

Spontaneous Inflation and the Origin of the Arrow of Time

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Abstract

We suggest that spontaneous eternal inflation can provide a natural explanation for the thermodynamic arrow of time, and discuss the underlying assumptions and consequences of this view. In the absence of inflation, we argue that systems coupled to gravity usually evolve asymptotically to the vacuum, which is the only natural state in a thermodynamic sense. In the presence of a small positive vacuum energy and an appropriate inflaton field, the de Sitter vacuum is unstable to the spontaneous onset of inflation at a higher energy scale. Starting from de Sitter, inflation can increase the total entropy of the universe without bound, creating universes similar to ours in the process. An important consequence of this picture is that inflation occurs asymptotically both forwards and backwards in time, implying a universe that is (statistically) time-symmetric on ultra-large scales.

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1 Introduction

The role of initial conditions in cosmology is unique within the physical sciences. We only have a single observable universe, rather than the ability to change boundary conditions and run experiments multiple times. A complete theory of cosmology therefore involves not only a set of dynamical laws, but a specification of the particular initial conditions giving rise to the universe we see.

One could certainly argue that the origin of our initial conditions is not an answerable scientific question. Given the state of our universe at the present time, and a complete set of dynamical laws describing its evolution, we can in principle solve for the entire history, including whatever the initial state was. Ultimately, we are stuck with the boundary conditions we have. Similarly, however, we are stuck with the laws of physics that we have, but this constraint doesn't stop us from searching for deep principles underlying their nature. It therefore seems sensible to treat our initial conditions in the same way, and try to understand why we have these conditions rather than some others.

In particular, we would like to know whether the initial conditions for our universe are in some sense “natural.” The notion of naturalness in this context is not precisely defined, but in simple cases we know it when we see it. Our current universe exhibits a pronounced matter/antimatter asymmetry, which seems unnatural, so we search for mechanisms of baryogenesis to create this asymmetry dynamically. The large-scale homogeneity and isotropy of our universe is unstable, so we search for mechanisms in the early universe to generate this smoothness dynamically.

One aspect of the boundary conditions for our universe that remains puzzling, or at least controversial, is the arrow of time (for discussions see [1, 2, 3]). The Second Law of Thermodynamics tells us that the entropy of a closed system is non-decreasing as a function of time. A comoving patch corresponding to our observable universe is not precisely a closed system, but is pretty close, since homogeneity of the universe near the boundary implies there should be no net entropy flux into or out of our patch. And indeed, we observe increasing entropy in our universe; the thermodynamic arrow of time is the direction picked out by this increase of entropy.

Long ago Boltzmann understood the increase of entropy as a statistical phenomenon, as evolution from a very small region of phase space to a larger region. The origin of the Second Law is traced back to initial conditions: the early universe had an extremely low entropy, allowing it to continue to increase thereafter. Even after 13.7 billion years of entropy growth, it remains low in our current universe. Within our observable patch, the entropy in ordinary matter is of order

$$S_M(U) \sim 10^{88} . \tag{1}$$

This figure is dominated by photons and neutrinos; dark matter may also contribute, but wouldn't significantly change the final answer. However, Penrose has emphasized [4] that this entropy could be much larger. Our universe remains quite smooth on large scales, which is a finely-tuned condition to be in. To see this, we can simply compare the current entropy to what it could have been, for example if more energy were stored in black holes. The

Bekenstein-Hawking entropy of a black hole [5, 6] is proportional to its horizon area,

$$S_{BH} = \frac{A}{4G} . \quad (2)$$

We believe that galaxies similar to ours contain supermassive black holes at their centers, of order a million solar masses; the entropy of a single such black hole is

$$S_{BH} \sim 10^{89} \left(\frac{M_{BH}}{10^6 M_\odot} \right)^2 . \quad (3)$$

Any one such black hole therefore contains more entropy than all of the ordinary matter in the visible universe. But there are perhaps 10^{10} such black holes in the universe; their total entropy is thus

$$S_{BH}(U) \sim 10^{99} , \quad (4)$$

which represents most of the entropy within our observable universe. This seems like a large number, but should be compared to how large it could be. The total amount of mass in the observable universe is

$$M_U \sim \rho H_0^{-3} \sim G^{-1} H_0^{-1} \sim 10^{22} M_\odot . \quad (5)$$

If all of this mass were collected into a single black hole, the entropy would be

$$S_{\max}(U) \sim 10^{121} , \quad (6)$$

larger than the actual entropy by a factor of 10^{22} . Moreover, in the early universe there weren't any black holes, and the entropy was dominated by the matter entropy (1), smaller than the equivalent black-hole entropy by a factor of 10^{33} . So the entropy of our universe seems to be very small, and seems to have evolved from an era where it was significantly smaller.

The initial conditions for our observable universe thus have extremely low entropy. We would like to understand these conditions as arising through dynamical evolution from a natural starting point; it is not immediately obvious how this could be achieved, however, since dynamical evolution tends to increase the entropy.

The low entropy of our current universe is clearly related to its homogeneity and isotropy, features which are supposed to be explained by the inflationary universe scenario [7, 8, 9, 10, 11]. It is therefore tempting to invoke inflation as the origin of the thermodynamic arrow of time, and indeed this move has been made [12]. The converse argument, however, has also been advanced: that inflation never works to decrease the entropy, and posits that our observable universe originates in a small patch whose entropy is fantastically less than it could have been, so that in fact there is a hidden fine-tuning of initial conditions implicit in the inflationary scenario, which consequently (in the extreme version of this reasoning) doesn't explain anything at all [4, 13, 14, 15].

In this paper we suggest a resolution of this apparent tension between inflation and the Second Law. We first discuss in general terms what sort of evolution for the universe would qualify as naturally giving rise to an arrow of time through dynamical processes. We then

review the arguments concerning the role of inflation in setting up the early universe in a low-entropy state, concluding that ordinary inflation does not directly explain the origin of the arrow of time if one believes in unitary evolution. We then argue that, in the absence of a vacuum energy or inflation of any sort, systems coupled to gravity generically evolve to flat, empty space. In the presence of a small positive vacuum energy (such as seems to exist in our universe [16, 17, 18]), systems will tend to initially empty out to an approximate de Sitter vacuum. If there is also at least one scalar inflaton field with an appropriate potential, however, there is an instability: quantum fluctuations in this field will eventually create an inflating patch, and according to the traditional arguments for eternal inflation [19, 20, 21, 22, 23] the total physical volume of inflating space will continue to increase thereafter, while “pocket universes” drop out of the inflating background and reheat. The idea of inflation beginning spontaneously has been suggested previously [24, 19, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34]. Recently, Garriga and Vilenkin [35] have suggested that spontaneous inflation in a low-energy de Sitter background could lead to a recycling universe, while Dyson, Kleban and Susskind and Albrecht and Sorbo have examined spontaneous inflation in the context of a causal-patch view inspired by the holographic principle [36, 37]; we discuss the relationship between our picture and these ideas in greater detail below.

The crucial features of this scenario are thus the following:

1. Generic initial states empty out and approach de Sitter space.
2. In the presence of an inflaton field, the de Sitter vacuum will be unstable to the onset of eternal inflation. Once eternal inflation starts, the entropy of the universe can grow without bound, never reaching thermal equilibrium.
3. In the process of increasing the entropy, eternal inflation creates large regions of space similar to our observable universe.

As an important consequence of our reasoning, we obtain a picture of the ultra-large-scale structure of the universe. As in any model of eternal inflation, the ultimate state of the universe is a fractal distribution of inflating and post-inflating regions [21, 22, 38, 39], in which our local Big Bang is not a particularly unique event. We also predict that this structure should be recovered infinitely far into the past, with a *reversed* thermodynamic arrow of time. Our overall universe is therefore statistically time-symmetric about some Cauchy surface of minimum entropy. The details of the state on this surface are not important; starting from a generic state, inflation eventually occurs towards both the past and future, erasing any information about the “initial” conditions.

2 Explaining the Arrow of Time Dynamically

Our goal is to understand how the arrow of time might arise from natural initial conditions for the universe. In this section we discuss some necessary features of any purported resolution of this issue.

To make any progress we need to have some understanding of what it means for an initial condition to be “natural.” If an unknown principle of physics demands specific initial

conditions (but not final ones), there is no problem to be solved; it may simply be that the universe began in a low-entropy initial state and has been evolving normally ever since. This might be the case, for example, if the universe were created “from nothing,” such that the initial state was *a priori* different from the final state [24, 25, 40, 26], or if Penrose’s explicitly time-asymmetric Weyl Curvature Hypothesis were true [4]. There is, of course, no way to rule out this possibility.

Nevertheless, it would be more satisfying if we could somehow understand our apparently low-entropy initial condition as an outcome of dynamical evolution from a generic state. In particular, we wish to avoid any explicit violation of time-reversal symmetry in the specification of the initial condition. The very word “initial,” of course, entails a violation of this symmetry; we should therefore apply analogous standards to the final conditions of the universe. Price [1] has proposed a “double-standard principle” – anything that is purportedly natural about an initial condition for the universe is equally natural when applied to a final condition. The conventional big-bang model clearly invokes unnatural initial conditions, as we would not expect the end state of a recollapsing universe to arrange itself into an extremely homogeneous configuration (as we discuss in more detail below).

Attempts to comply with the double-standard principle (or its underlying philosophy) have occasionally led to proposals for explicitly time-symmetric cosmologies in which entropy is low both at the beginning and the end of time. In 1962, Gold proposed a recollapsing universe with similar conditions imposed at the Bang and Crunch [41, 1]. In the Gold universe, entropy increases as the universe expands, reaches a maximum, and then begins to decrease again. Hawking at one time suggested that such behavior could follow naturally from quantum cosmology, but Page argued persuasively that it was not necessary [42, 43]. More recently, Gell-Mann and Hartle have investigated the possibility of imposing quantum-mechanical boundary conditions in both the past and future [44]. It might be difficult to find a consistent implementation of the Gold universe, and Hartle and Gell-Mann even suggest that the possibility of a time-symmetric Bang and Crunch may be ruled out by observation (from the absence of observed backward-moving radiation emitted by stars in the future). For most people, the important objection to these models is simply that they seem hopelessly *ad hoc*; imposing boundary conditions both in the past and the future would appear to be the opposite of “natural.” Price [1] argues that imposing low-entropy conditions in the past and the future is the only way to avoid conflict with the double-standard principle. We will suggest a loophole in this argument, by suggesting that *high*-entropy conditions obtain in the far past and far future, and reconciling this unusual claim about the past by arguing that our observed early universe is merely a small part of a larger state.

One way to understand the concept of naturalness would be to imagine that we had a measure on the space of microstates describing the universe, and an appropriate way of coarse-graining into macroscopically indistinguishable states. The entropy is then simply the logarithm of the measure on any collection of macroscopically indistinguishable microstates, and natural initial conditions are those with large entropy. Unfortunately, we do not have a reliable understanding of entropy in the context of quantum gravity; the fundamental degrees of freedom themselves remain unclear. Even without a quantitative understanding at the level of statistical mechanics, however, there are certain essential features that entropy must have, which will be sufficient to highlight problems with the conventional picture.

In ordinary thermodynamics, the fundamental property of the entropy is that it tends to increase (or at least not decrease) with time. Of course, this property is understood as a consequence of low-entropy initial conditions. If we imagine choosing initial conditions randomly, with uniform probability for choosing any microstate, a system with a finite number of degrees of freedom will typically be found in a state of maximum entropy. Maximum-entropy states represent thermal equilibrium, and are generally static (at least macroscopically). Statistical fluctuations will allow the entropy to occasionally, although rarely, decrease from that point, and then return to equilibrium. In thinking about the early universe, it is worth keeping in mind that a state that rapidly evolves into something else cannot possibly be a “natural” state in the sense of having maximal entropy, since natural states should be static (at least statistically).

Given an infinite amount of time, a finite system will approach arbitrarily close to all possible configurations consistent with the existence of certain conserved quantities (Poincaré recurrence). It is therefore irresistible to ask whether our observed arrow of time could arise as the result of such a fluctuation. The answer seems to be “no,” for the basic reason that our observed universe has a much lower entropy than can be explained in this way. For example, if we appeal to the anthropic principle to understand our current state, we would only require a fluctuation in entropy large enough to allow for a single conscious observer, not an entire universe (“Boltzmann’s Brain” [45, 2, 37]). Further, even if we consider the set of universes which are similar to ours, most of them are not thermodynamically sensible, in the sense of arising from lower-entropy initial conditions [36]. It therefore requires more than a simple fluctuation out of thermal equilibrium to explain our current state. (In the context of causal-patch physics this problem becomes particularly acute, as there really are only a finite number of degrees of freedom to be considered. Albrecht and Sorbo have argued that inflation can overcome the Boltzmann’s brain paradox even in this context [37]; as we discuss in Section 3.2.3, we are skeptical that any theory of fluctuations around equilibrium would be able to resolve this puzzle.)

We are left with the following conundrum: we would like to explain our currently observed universe as arising from natural initial conditions, but natural means high-entropy, and high-entropy implies equilibrium configurations with occasional fluctuations, but not ones sufficient to explain our observed universe. However, there is one loophole in this reasoning, namely the assumption that there is such a thing as a state of maximal entropy. If the universe truly has an infinite number of degrees of freedom, and can evolve in a direction of increasing entropy from any specified state, then an explanation for the observed arrow of time arises more naturally.

The situation is analogous (although not equivalent) to the behavior of a particle moving in a potential that rolls off to infinity without having any minima, such as $V(x) = 1/x$, as portrayed in Figure 1. A free particle in such a potential comes in from infinity, reaches a turning point, and returns to infinity. On any such trajectory, every point except the turning point is either moving towards, or coming from, larger values of x . Now imagine that entropy in the universe behaves the same way. It would not be surprising to find ourselves in a situation where the entropy were evolving; it would be almost inevitable, and certainly perfectly natural. Indeed, we would simply *define* the “past” in such a universe to be the direction of time in which the entropy was *locally* decreasing. All that is needed to

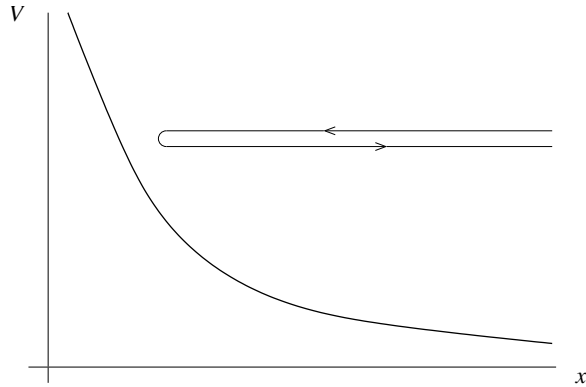


Figure 1: Motion of a particle in a potential that rolls to infinity without ever reaching a minimum. All trajectories in this potential come in from infinity, reach a turning point, and return to infinity.

have an arrow of time arise dynamically is for the entropy to be unbounded above, so that it can always increase from any given starting point.

In Figure 2 we portray the various possible ways to address the arrow-of-time problem, in terms of the evolution of entropy as a function of time. The first choice is to impose time asymmetry by hand, simply insisting on low-entropy initial conditions; this scenario is plausible, but we would prefer not to rely on an *ad hoc* initial condition. The second is to impose low-entropy conditions at both initial and final times, as in the Gold universe. Such a picture is also possible, but equally *ad hoc*, as it implies that the universe is actually finely-tuned at every possible moment (to be consistent with the future boundary condition). The third possibility is to imagine that the early universe is a random low-entropy fluctuation from an equilibrium state; this seems implausible on anthropic grounds. The final possibility, advocated here, is that there is no equilibrium configuration for the universe; instead, the entropy can increase without bound, and will generally do so from any given boundary condition. (Such a boundary condition is not actually imposed at a boundary, but at some arbitrary moment.) We should stress that there might also be, and perhaps should be, some procedure for actually specifying this boundary condition; but according to our picture this specification will leave no observable imprint on our universe, and in that sense is truly irrelevant.

Of course, it is certainly not *sufficient* to imagine that the entropy of the universe is unbounded, although it seems to be necessary. We also need to understand why the process of entropy creation would create regions of spacetime resembling our observable universe. This is where we can sensibly appeal to inflation, as discussed in the next section.

3 Inflation and its Discontents

Inflation is an extremely powerful and robust idea. It purports to solve several severe fine-tuning problems of conventional cosmology, including the horizon, flatness, and monopole

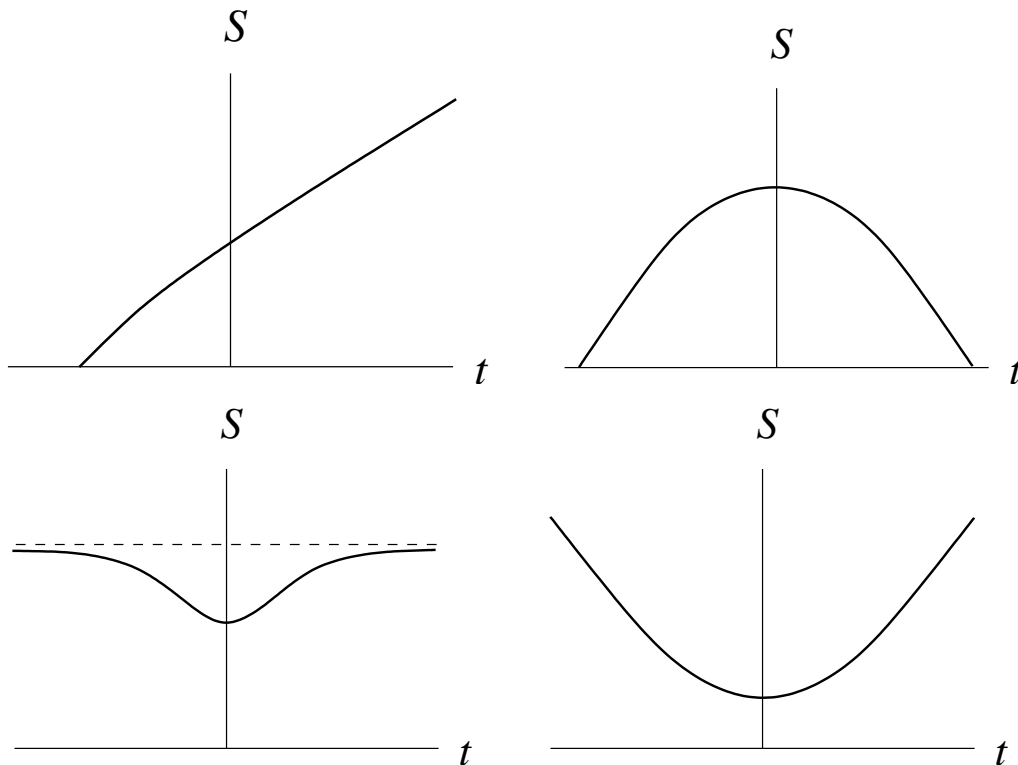


Figure 2: Possible evolutions of the entropy of the universe. At top left, a universe that comes into existence with prescribed low-entropy conditions in the past; at top right, a universe with low-entropy conditions in both the past and future. At bottom left, a fluctuation from an equilibrium value of maximum entropy, portrayed by the dashed line. At bottom right, a universe with unbounded entropy to the past and future.

problems. As a bonus, it provides a natural mechanism for the origin of the approximately scale-free density perturbations that serve as the origin of structure in our universe. Perhaps equally importantly, it provides a natural explanation for why the observable universe should be large and expanding at all, as inflation can create a fantastic number of particles from a microscopically small initial patch of spacetime.

However, there are some lurking problems in our understanding of the onset of inflation and the role of initial conditions. In this section we review some of the existing controversies over initial conditions for inflation and the role of entropy, arguing as effectively as we can that there really is a serious issue that has yet to be addressed concerning the onset of inflation. This discussion will set the stage for our suggested resolution of this problem in later sections.

3.1 Starting Inflation

Let's review the standard picture of inflation, choosing for definiteness a simple quadratic potential

$$V(\phi) = \frac{1}{2}m^2\phi^2, \quad (7)$$

where the scalar field ϕ is the inflaton, and, in order to get the correct amplitude of density perturbations, the mass parameter is chosen to be

$$m \sim 10^{13} \text{ GeV}. \quad (8)$$

The Friedmann equation can be written

$$H^2 = \frac{8\pi}{3M_{\text{pl}}^2}\rho, \quad (9)$$

where the ordinary Planck mass is $M_{\text{pl}} = 1/\sqrt{G} \sim 10^{19} \text{ GeV}$. In this model, inflation can occur when the field value is above a certain critical value

$$\phi_I \sim M_{\text{pl}}, \quad (10)$$

so that the energy density ρ_I is near the scale of grand unification $\rho_I \sim (10^{16} \text{ GeV})^4$, and the Hubble radius about 10^6 times the Planck length,

$$H_I^{-1} \sim (10^{13} \text{ GeV})^{-1} \sim 10^6 L_{\text{pl}}. \quad (11)$$

(This is the requirement for inflation with an appropriate amplitude of density fluctuations; eternal inflation only occurs at a higher scale, as we discuss later.) In order for inflation to begin, there must be a region that is smooth, dominated by potential energy, and of radius L_I larger than H_I^{-1} [46, 47, 48, 49]. If the initial value of the scalar field is sufficiently large, inflation will occur for more than the sixty e -folds of accelerated expansion necessary to bring our entire currently observable universe into causal contact. At the end of the inflating period, we imagine that the energy of the inflaton is converted by reheating into excitations of ordinary matter and radiation.

During the period of accelerated expansion, the spatial curvature of the initial patch is driven to zero, and any small inhomogeneities diminish as the universe rapidly stretches. In this way, a very small proto-inflationary region can naturally evolve into our observable patch today, explaining its high degree of homogeneity and isotropy. Thus, although smoothness and flatness are unstable features in conventional cosmology, inflation drives the universe towards such a state, thereby (so the story goes) explaining these otherwise puzzling features.

Inflation thus claims to provide a mechanism by which a universe like ours can arise robustly from random initial conditions, not requiring fine-tuning to preserve homogeneity and isotropy. As mentioned in the introduction, a simple alternative would simply be that the universe did not start with random initial conditions, but rather with the appropriate ones to evolve straightforwardly into our present state. In the absence of some specific theory of initial conditions that would actually predict such an initial state, this latter point of view

doesn't actually explain anything; inflation, in contrast, purports to explain why our universe should look the way it does almost independently of the actual initial conditions. However, the strength of this explanation relies heavily on a certain intuitive notion of what constitutes a "random" or "generic" state. In particular, there is an assumption that it is more likely to find an appropriate proto-inflationary patch than it is to find a patch resembling an early stage of the conventional hot Big Bang model.

Some specific arguments have been advanced that it cannot be *too* unlikely to find a patch of space with appropriate proto-inflationary conditions lurking within a randomly-chosen initial state. Consider, for example, the allowed modes of a field confined to a small patch of linear size L_I . Since this patch is only slightly larger than the Planck length, there are only a certain number of modes with sub-Planckian energies [perhaps $(L_I/L_{\text{pl}})^3 \sim 10^{18}$]; we need only to have most of these modes be in their ground states in order for inflation to begin. Another version of the argument comes from Linde's chaotic inflationary scenario [50], in which randomly fluctuating conditions in the early universe will occasionally give rise to an appropriate patch that begins to inflate; once inflation does start, the fantastic increase in the volume of space guarantees that most of the universe will eventually be found in the inflationary or post-inflationary regions. Indeed, inflation could plausibly begin in a region of a single Planck volume with an energy density at the Planck scale; all we need is for a fluctuation to give rise to one such region [51].

3.2 The Unitarity Critique

A critique of the conventional inflationary scenario has been advanced, in slightly different versions, by Penrose [4], Unruh [14], and Hollands and Wald [15], and in the context of causal-patch physics by Dyson, Kleban and Susskind [36]. The claim of these counterarguments is essentially that it is extremely unlikely to find a patch of space in the appropriate proto-inflationary conditions – less likely, even, than to find the universe in the initial conditions for the conventional Big Bang model. We will first present this argument in two similar forms: one using entropy as a measure on the space of initial conditions, and one appealing to our notions of what would be likely for a collapsing universe. We will then discuss the closely related issues raised by the causal-patch picture suggested by holography, which implies that one should consider only finite number of degrees of freedom contained in each de Sitter volume.

3.2.1 Entropy version

The most straightforward and dramatic version of this argument involves comparing the entropy of the early universe to that of the universe today [4]. The entropy is the logarithm of the volume corresponding to macroscopically indistinguishable states, and therefore measures the likelihood of "randomly" choosing a certain condition, if we assign uniform measure to each microscopic configuration. (This is, of course, a nontrivial assumption. In particular, a universe that fluctuates into existence out of nothing may do so into a very specific state, not a randomly-chosen one.) The crucial point here is that our currently observable universe and

the small patch of early universe that grows into it are not two different physical systems; they are two different *configurations* of the *same system*.

This fact seems at once completely obvious and deeply counterintuitive. Of course the small patch in the early universe and the currently observable patch into which it grows are two configurations of the same system; after all, one evolves into the other. On the other hand, they appear very different – different volumes, different total energies, different numbers of particles. It is therefore tempting to judge them according to different sets of standards, while in fact they simply represent different subsets of the state space of (the comoving volume corresponding to) our universe. This is a consequence of the fundamental weirdness of statistical mechanics in the presence of gravity, about which we will have more to say in the next section.

Let us then consider the entropy of the proto-inflationary patch. We don't have a reliable way to calculate this entropy, but there are reasonable guesses, and the precise answer is not important. Since the patch is taken to be in a quasi-de Sitter initial state, we could, for example, consider the corresponding de Sitter entropy for a patch of that size,

$$S_I \sim \left(\frac{L_I}{L_{\text{pl}}} \right)^2 \sim 10^{12} . \quad (12)$$

Alternatively, we could simply count the number of Planck volumes in the proto-inflationary region, $(L_I/L_{\text{pl}})^3 \sim 10^{18}$, imagining that there is of order one degree of freedom per Planck volume. Of course, the proper measure of the entropy might be infinite, if there are truly an infinite number of degrees of freedom; but in that case the entropy of the late universe is also infinite, and the numbers given here are an appropriate standard for comparison.

The point of the entropy argument is then very simple: the entropy of the patch that begins to inflate and expands into our observable universe is far less than our current entropy (4), or even than the entropy at earlier stages in our comoving volume, given approximately by the matter entropy (1). From the point of view of the Second Law, this makes sense; the entropy has been increasing since the onset of inflation. But from the point of view of a theory of initial conditions, it strongly undermines the idea that inflation can naturally arise from a random fluctuation. In conventional thermodynamics, random fluctuations can certainly occur, but they occur with exponentially smaller likelihood into configurations with lower entropy. Therefore, if we are imagining that the conditions in our universe are randomly chosen, *it is much easier to choose our current universe than to choose a small proto-inflationary patch*. The low entropy of the proto-inflationary patch is not a sign of how natural a starting point it is, but of how extremely difficult it would be to simply find ourselves there from a generic state. If inflation is to play a role in explaining the initial conditions of the universe, we need to understand how it arises from some specific condition, rather than simply appealing to randomness.

3.2.2 Reversibility version

The reversibility version of the unitarity critique does not explicitly use the entropy as a measure on the space of randomly chosen initial states, but instead attempts to characterize

what we should consider a “natural” starting condition by implicitly invoking the double standard principle and examining what we would consider natural final conditions for a collapsing universe [14, 15].

Even in the absence of a reliable measure on the space of initial conditions, it seems sensible to invoke Liouville’s theorem, which states that dynamical evolution preserves the measure on phase space. In other words, given a measure μ on the phase space of a system, and two subspaces A_i and B_i representing two different classes of initial conditions that evolve into final conditions A_f and B_f , we will have

$$\frac{\mu(A_i)}{\mu(B_i)} = \frac{\mu(A_f)}{\mu(B_f)} . \quad (13)$$

If A_i represents initial conditions that are “unlikely” and B_i represents initial conditions that are “likely,” they evolve into final conditions A_f and B_f that are unlikely and likely, respectively.

Assuming Liouville’s theorem holds, the reversibility critique is simply the statement that, within the set of initial conditions that evolve into a universe like ours, the subset that does not pass through an inflationary phase is much larger than the one that does. It is easiest to see this by starting with our present universe considered in a coarse-grained sense (*i.e.*, the space of all microstates that macroscopically resemble our current universe). If we take states in this set and evolve them backwards in time, or equivalently consider collapsing universes with conditions otherwise similar to our own, we generically expect an increasing departure from homogeneity as space contracts and gravitational perturbations grow. Black holes will tend to form, and the approach to a singularity will be highly non-uniform, little resembling the smooth Big Bang of our actual past history. Certainly it seems extremely unlikely that the universe would smooth out and “anti-inflate,” with a period of re-freezing in which hot matter cleverly assembles itself into a smooth inflaton field which then rolls up its potential into a contracting quasi-de Sitter phase.

The reversibility critique is thus very simple: the measure on the space of conditions that anti-inflate in a contracting universe is a negligible fraction of that on the space that collapse inhomogeneously (or even relatively homogeneously, but without anti-inflation); but the ratio of these measures is the same as the ratio of the measure on the space of inflationary initial conditions to that on the space of all conditions that evolve into a universe macroscopically like ours. Therefore, if our initial conditions are truly chosen randomly, inflationary conditions are much less likely to be chosen than some other conditions that evolve into our universe (which are still a tiny fraction of all possible initial conditions). In other words, nothing is really gained in terms of naturalness by invoking an early period of inflation; as far as random initial conditions go, it requires much less fine-tuning to simply put the universe in some state that can evolve into our present conditions.

The idea that our current universe is more likely to be randomly chosen than a small, smooth patch dominated by a large potential energy seems intuitively nonsensical. Our current universe is large, complicated, and filled with particles, while the proto-inflationary patch is tiny, simple, and practically empty. But our intuition has been trained in circumstances where the volume, or particle number, or total energy is typically kept fixed, and none of these is true in the context of quantum field theory coupled to gravity.

3.2.3 Causal-patch version

An interesting perspective on the likelihood of different initial conditions for the universe arises from the causal-patch description of spacetime [52, 53, 54, 55].¹ This view derives from two profound lessons of black-hole physics: the holographic principle [59, 60, 61] and black-hole complementarity [62]. A concrete consequence of the holographic principle is Bousso’s covariant entropy bound, which places a limit on the entropy that can be contained within a region [63]. More specifically, given a spacelike region Σ with boundary $\partial\Sigma$, the amount of entropy that can pass through an appropriately constructed inward-pointing null surface emanating from the boundary is given by the area of $\partial\Sigma$ as measured in Planck units. Black-hole complementarity, meanwhile, suggests that consistent descriptions of physical situations may only include events within the horizon of a single observer. In the context of a de Sitter spacetime, these two ideas lead to the causal patch perspective, which states that we should only consider the physics inside a single causal patch bounded by the de Sitter horizon. Since the area of such a horizon is finite (with radius $R_{\text{dS}} = \sqrt{3/\Lambda}$), the causal patch has only a finite number of degrees of freedom.

Once we imagine that there are only a finite number of degrees of freedom, the lessons of Poincaré recurrence discussed in Section 2 become relevant. There are only a finite number of configurations of the causal patch, and over an infinite period of time every possibility will be sampled infinitely often. The equilibrium configuration is empty de Sitter, but universes like our own can arise through statistical fluctuations, either directly or via a period of inflation. Unfortunately, as discussed above, it is exponentially easier to fluctuate into a universe which is anthropically allowed but thermodynamically nonsensical than into a universe that evolved normally from an extremely low-entropy initial state.

Dyson, Kleban, and Susskind [36] have forcefully emphasized this apparent contradiction between causal-patch physics and thermodynamics. A possible reconciliation has been suggested by Albrecht and Sorbo [37]. They argue that inflation can be favored over ordinary evolution, even if there are only a finite number of degrees of freedom, by calculating the entropy of the proto-inflationary patch to include the entropy of the larger de Sitter region in which it is embedded. They suggest that this preference for inflation can resolve the “Boltzmann’s Brain” paradox, since inflation leads to a large-volume universe with many brains. There is an apparent tension, however, between this point of view and the reversibility critique of inflation. In a state of thermal equilibrium, any one kind of fluctuation should be exactly equally as likely as its time-reversed counterpart. We can certainly imagine a fluctuation in which a contracting universe is gradually assembled, smoothes itself out, and anti-inflates; but for the same reasons given in the previous subsection, this seems much less likely than contraction to a highly inhomogeneous Big Crunch. It seems very difficult to derive a true arrow of time from fluctuations about an equilibrium background.

Our perspective is quite different, in that we imagine there are truly an infinite number of degrees of freedom, and that we may sensibly speak of causally disconnected parts of the universe. For us, de Sitter is not an equilibrium state about which we are fluctuating,

¹An alternative implementation of causal-patch physics is the “holographic cosmology” of Banks and Fischler [56, 57, 58]. This scenario offers a very different perspective on the dynamics of the early universe, which we do not consider in this paper.

but a metastable state that eventually decays via spontaneous inflation, as discussed below. Strictly speaking, these assumptions are not inconsistent with the covariant entropy bound. The initial proto-inflationary patch must be somewhat larger than the inflationary Hubble radius; if such a spacetime remained de Sitter for all time, the corresponding light-sheets emanating inward from the boundary would not form a closed surface, and the holographic bound places no constraint on the number of degrees of freedom inside. One might attempt to subdivide the patch into smaller regions and calculate the bound for each sub-region, but that would only make sense if the bound were additive for nearby regions (*i.e.*, if the entropy flux were localized into individual regions). In other words, such a subdivision neglects the entropy corresponding to the relative configurations of neighboring patches, which can presumably be important.

Of course, the proto-inflationary patch does not remain in a quasi-de Sitter phase forever, but eventually reheats and the Hubble parameter begins to decrease. In a spatially flat universe with zero cosmological constant, the light-sheets would ultimately close and the holographic bound would be finite. It is therefore amusing to note that the current acceleration of the universe implies that we are once again entering a de Sitter phase, and the light-sheets emanating from the boundary of the original proto-inflationary patch lie outside our de Sitter horizon. Thus, they will again not form a closed surface (although the situation can change if the dark energy is quintessence rather than a cosmological constant [64, 65]). We do not know whether there is some deep connection between inflation, holography, and the current value of the vacuum energy. For the rest of this paper, in any event, we will simply assume that a semiclassical treatment with an infinite number of degrees of freedom is valid.

3.3 Unitarity and Degrees of Freedom

In any version of this critique of inflation, there is a crucially important assumption: that the evolution of the comoving volume from the early universe to today is unitary (and hence reversible). The assumption of unitarity seems innocuous, but is extremely profound in this context; it underlies the claim that the small proto-inflationary patch and our current universe are the same system, just in two different configurations.

An obvious prerequisite for unitary evolution is the conservation of the set of degrees of freedom characterizing the system. Such an assumption is so deeply ingrained in our understanding of dynamics that it is not usually spelled out explicitly, but in the case of an expanding spacetime the issue is not so clear. Indeed, unitarity is directly at odds with the reasoning behind the idea that it is much easier to begin inflation than to simply find the universe in its current state: the assertion that there are not that many degrees of freedom that need to be in their vacuum state in order for a small region to begin inflating. If the evolution is truly unitary, there are a fantastically large number of such degrees of freedom, most of which would correspond in a Fourier decomposition to modes with frequencies well above the Planck scale, and each of these degrees of freedom needs to be separately put into its vacuum state. These modes would generically be excited in a time-reversed or collapsing universe, as inhomogeneities grew and black holes were formed. In other words, the idea that degrees of freedom are neither created nor destroyed dramatically undercuts the hope that

it is not too unlikely to find a patch of the universe in an appropriately proto-inflationary configuration.

It is by no means obvious that the evolution should be unitary. A well-known idea of Hawking (recently recanted) suggests that unitarity is violated in the process of black-hole evaporation [66]; it is equally reasonable to imagine that other processes involving semi-classical quantum gravity are also non-unitary, including inflationary cosmology. Kofman, Linde, and Mukhanov [51] have pointed out that there is an apparent violation of unitarity in the process of reheating, due to particle production. Whether one chooses to think of this process as truly non-unitary is to some extent a matter of choice, depending on one's point of view toward collapse of the wavefunction; from a Copenhagen point of view it is truly non-unitary, while from a many-worlds perspective the evolution of the entire wavefunction is perfectly unitary. (See also the response by Hollands and Wald [67], and the paper by Mathur [68].) If unitarity is violated, degrees of freedom are brought into existence as the universe expands, so that our current universe has a much larger number of degrees of freedom than were present in the corresponding comoving volume at the beginning of inflation. A violation of unitarity is a dramatic possibility, but it could conceivably vitiate the critique of inflation just discussed; if there are very few degrees of freedom initially in the system, the initial state may have an entropy that is nearly maximal even if it is much lower than the entropy today.

Our perspective in this paper will be to maintain unitarity, but instead examine carefully the notion of a generic state in a theory with gravity. If the beginning of inflation relies on a random fluctuation from chaotic initial conditions, it is unclear what is gained by invoking a fluctuation to a configuration with such an unnecessarily low entropy. In the next section we will argue that the conventional picture of “generic initial conditions” is misleading. Rather than a state with large fluctuations at or near the Planck scale, we will suggest that a generic state is close to empty space. The entropy density of such a configuration will be very low, but the total entropy will be very large. From this specific kind of configuration, in the presence of a positive vacuum energy, quantum fluctuations of an inflaton field can lead to the spontaneous onset of inflation, which then proceeds normally, eventually reheating and creating regions similar to our observed universe.

4 Typical States in Theories with Gravity

A textbook example of the approach to thermal equilibrium from a low-entropy state is the evolution of a gas of particles in a box, as portrayed in Figure 3. If the gas is originally located in some small corner of the box, it will typically evolve toward a higher-entropy state by spreading throughout the box. The state of maximum entropy will represent thermal equilibrium, in which the coarse-grained state (described in terms of macroscopic observables rather than the microscopic information about each molecule of the gas) remains static.

As Penrose has emphasized [4], this story is different once we turn on gravity. (In this section we imagine that the vacuum energy is zero, so that empty space is flat.) If the size

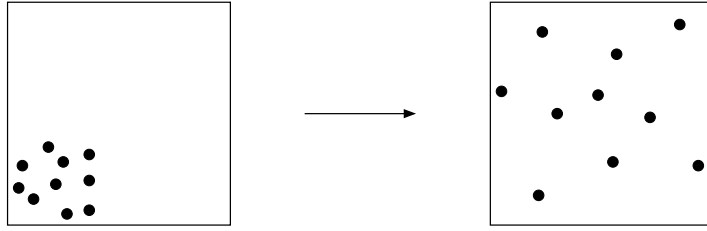


Figure 3: In the absence of gravity, a box of gas evolves to an equilibrium state with a homogeneous distribution.

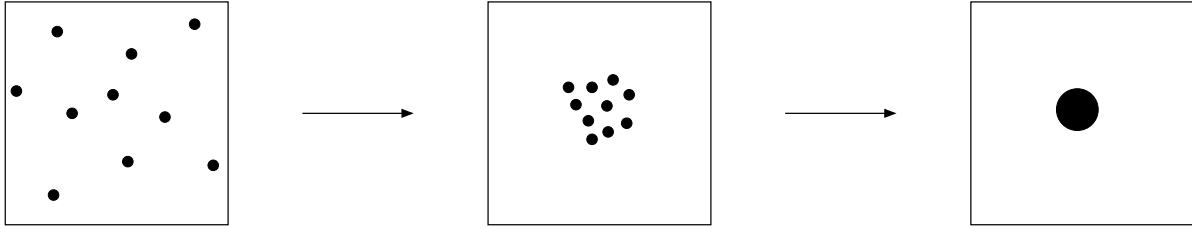


Figure 4: In the presence of gravity, a box of gas with size greater than the Jeans length evolves to form a black hole via gravitational instability.

of the box is greater than the Jeans length

$$\lambda_J = \sqrt{\frac{\pi v_s^2}{G\rho}}, \quad (14)$$

where v_s is the sound speed and ρ the energy density, the gas will be unstable to gravitational clumping, as shown in Figure 4. The tendency will therefore *not* be for the gas to become homogeneous, but rather concentrated in a localized overdense region. Given enough time, the overdense region should form a black hole; even if the equation of state of the gas supports a metastable compact object (such as a planet or degenerate star), there will be some amplitude for a quantum fluctuation to collapse the object to a black hole. It is therefore clear, even in the absence of a reliable theory of gravitational entropy, that the compact objects have a greater entropy than the homogeneous gas; whatever entropy is, it tends to increase in the course of typical evolution from a state with non-maximal entropy.

Although the black hole is higher entropy than the original homogeneous gas, it cannot be a maximum-entropy state, since it is not static; the black hole will gradually evaporate by emitting Hawking radiation.² Although we could imagine constructing a sturdy box with reflecting boundary conditions in such a way that the black hole eventually comes into equilibrium with the radiation in the box, such a configuration is clearly contrived and unstable; any rupture in the integrity of the box would allow the radiation to escape, and

²It is often stated that “black holes are configurations of maximum entropy,” but this is true only when the area of the boundary is held fixed. In a realistic context where spacetime evolves and areas are not fixed, black holes do not maximize the entropy. (If they did, they wouldn’t evolve into something else.) Stable radiating black holes may exist in anti-de Sitter space, but that is not the real world.

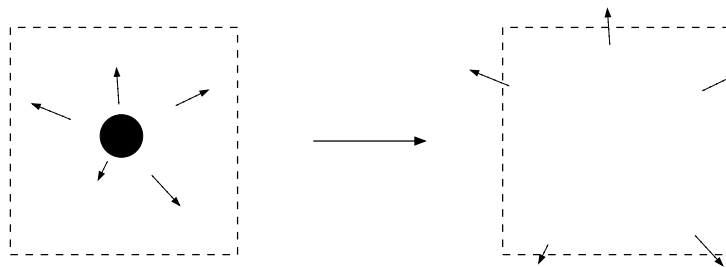


Figure 5: With asymptotically flat boundary conditions, the black hole evaporates away, leaving a thin gas of radiation in an increasingly empty space.

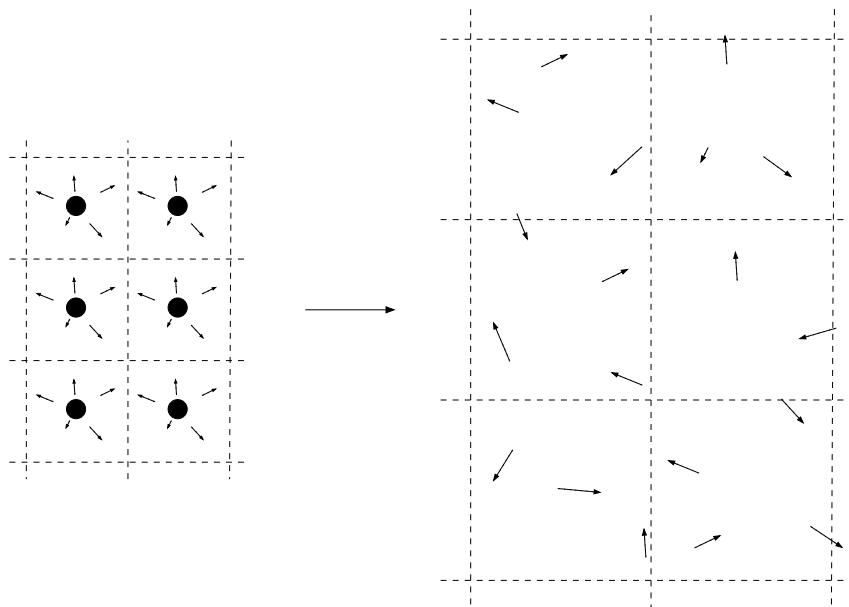


Figure 6: In an expanding universe, black holes evaporate and the resulting radiation becomes increasingly rarefied, so that the universe approaches flat spacetime.

there will be some small probability that the entire box will tunnel into the black hole. Instead, let us take the box itself to represent some simple set of boundary conditions, and ask what the ultimate state of the system might be. There are innumerable possible conditions that could pertain outside the box, but it is useful to consider two limiting cases: an asymptotically-flat empty universe, and periodic boundary conditions.

If the universe is empty outside the fictitious box, it is obvious what happens: the black hole evaporates away, and the ultimate state is nearly-flat empty space, as the Hawking radiation disperses to infinity as shown in Figure 5. One can prove [69, 70, 71, 72] certain versions of the Generalized Second Law, which guarantees that the radiation itself, free to escape to infinity, does have a larger entropy than the original black hole. (For the long-term fate of objects in an astrophysical context, see [73].)

The case of periodic boundary conditions is more subtle. Such a configuration describes a

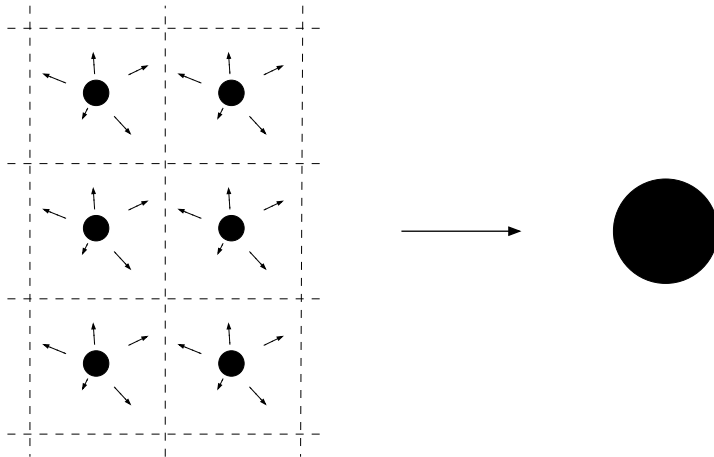


Figure 7: A contracting universe filled with black holes collapses to a singularity. If the “universe” is only a compact piece of a larger space, the singularity will be a black hole that can radiate away.

lattice of black holes throughout space, equivalent on large scales to a cosmological spacetime with a certain average energy density. Einstein’s equation then tells us that the overall scale factor of the space must be expanding or contracting. If it is expanding, the black holes will be able to evaporate away, leaving a universe of gradually diluting radiation, ultimately approaching flat spacetime, as shown in Figure 6. We are therefore left with a final state similar to the asymptotically-flat case of the previous paragraph.

If the scale factor is contracting, however, the density will increase until the universe hits a Big Crunch singularity in the future, as shown in Figure 7. But we can argue that collapse to a cosmological singularity is a non-generic situation. More specifically, specifying initial data in any region with compact support is never sufficient to guarantee a future cosmological singularity. (We are assuming that the universe is non-compact; if spatial sections are compact, Big Crunch singularities can be generic.) Given initial data in some local region describing a collapsing universe, we can always surround the region by a large but finite region of empty space. Now the collapse actually describes a black hole, which eventually radiates away as before. To ensure that the entire universe collapses to a final singularity requires that we specify initial data over the entire infinite spacelike hypersurface; we therefore conclude that such behavior is non-generic, and evolution to flat empty space is the only robust outcome. (This conclusion is not absolutely necessary for our general picture; if the argument against future singularities is unconvincing, simply replace “empty space” in subsequent claims by “empty space or a future singularity.”)

The examples considered in this section provide anecdotal evidence for a straightforward claim: in a theory with gravity (and vanishing vacuum energy), the closest thing to a maximal-entropy, thermal-equilibrium state is flat empty space. Another way to reach the same conclusion is to simply take any configuration defined on a spacelike hypersurface, and realize that we can increase the entropy by taking the same set of excitations (whether in matter fields or gravitational waves) and spread them apart by increasing the scale factor,

thereby increasing the allowed phase space for each particle. This expansion doesn't violate any conservation laws of the system, so there is no obstacle to configurations eventually increasing their entropy in this fashion.

Consequently, there is no reason to expect randomly-chosen or generic conditions to feature large curvatures and Planck-scale fluctuations. According to everything we know about gravity, large curvatures are entropically disfavored, tending to ultimately smooth themselves out under ordinary evolution. This is a direct consequence of the ability of a curved spacetime to evolve towards perpetually higher entropy by having the universe expand, unlike gas trapped in a box. From this point of view, it should not be considered surprising that we live in a relatively cold, low-curvature universe; the surprise is rather that there is any observable matter at all, much less evolution from an extremely hot and dense Big Bang.

The idea that high-entropy states correspond to nearly-flat empty space makes the conundrum of the initial conditions for inflation seem even more acute – such a condition seems very different from the chaotic initial conditions often invoked in discussions of inflation. In the next section we explore how inflation can originate from quantum fluctuations in a nearly-empty universe with a small positive cosmological constant and an appropriate inflaton field.

5 Spontaneous Inflation from Cold de Sitter Space

In the previous section we argued that generic initial conditions in a theory with gravity tend to evolve to flat empty space, which is correspondingly the highest-entropy state in the theory. (The entropy *density* is low, but it is the total entropy which tends to increase according to the Second Law.) If initial conditions for the universe are randomly chosen, with high-entropy states correspondingly preferred, the natural question is then why we observe any matter in the universe at all, much less an extremely dense Big Bang.

A way out of this conundrum is possible if “empty space” is not a perfectly stable state, but rather is subject to instabilities that can produce universes like our own. A mechanism for such an instability may be provided by the small positive cosmological constant that has apparently been discovered [16, 17, 18]. (If the acceleration of the universe is due to dynamical dark energy or a modification of gravity on large scales, the discussion to follow would change in important ways and perhaps become irrelevant.)

The basic idea is very simple. In the presence of a positive vacuum energy, it will remain true that most states tend to empty out to empty space, but “empty space” will correspond to de Sitter rather than Minkowski spacetime. Unlike Minkowski, which has zero temperature, and a de Sitter space with vacuum energy density $\rho_{\text{vac}} = M_{\text{vac}}^4$ will have a Gibbons-Hawking temperature

$$T_{\text{dS}} = \frac{H}{2\pi} \sim \frac{M_{\text{vac}}^2}{M_{\text{pl}}} . \quad (15)$$

This temperature gives rise to thermal fluctuations in any fields in the theory. In this section, we describe how fluctuations in an appropriate inflaton field ϕ can lead to the spontaneous onset of inflation, which can then continue forever as in the standard story of

eternal inflation. This idea is not new; Garriga and Vilenkin, for example, have proposed that thermal fluctuations can induce tunneling from a true de Sitter vacuum to a false vacuum at higher energies, thus inducing spontaneous inflation [35]. (There is also a body of literature that addresses the creation of inflating universes via quantum tunneling, either “from nothing,” at finite temperature, or from a small patch of false vacuum [24, 19, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34].) In our discussion is that we examine the case of an harmonic oscillator potential without any false vacua; in such a potential we can simply fluctuate up without any tunneling. The resulting period of inflation can then end via conventional slow-roll, which is more phenomenologically acceptable than tunneling from a false vacuum (as in “old inflation” [7]). Thus, the emptying-out of the universe under typical evolution of a generic state can actually provide appropriate initial conditions for the onset of inflation, which then leads to regions that look like our universe.

We should emphasize that our calculation ignores many important subtleties, most obviously the back-reaction of the metric on the fluctuating scalar field. Nevertheless, our goal is to be as conservative as possible, given the limited state of our current understanding of quantum gravity. In particular, it is quite possible that a similar tunneling into inflation may occur even in a Minkowski background (see *e.g.* [34]). In our calculation we simply discard the vacuum fluctuations that are present in Minkowski, and examine only the additional contributions from the nonzero de Sitter temperature. We believe that the resulting number (which is fantastically small) provides a sensible minimum value for the probability to fluctuate up into inflation. The true answer may very well be bigger; for our purposes, the numerical result is much less important than the simple fact that the background is unstable to the onset of spontaneous inflation. Clearly this issue deserves further study.

5.1 Eternal Inflation

We first recall the basics of eternal inflation [19, 20, 21, 22]. As in Section 3, consider a massive scalar field with an potential $V(\phi) = \frac{1}{2}m^2\phi^2 + V_0$ and a mass $m \sim 10^{13}$ GeV. (This example is merely illustrative; the details of the potential are not crucial, so long as some simple requirements are met.) Classically, the field will roll down the potential according to

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0 . \quad (16)$$

In the slow-roll regime, where we can ignore the $\ddot{\phi}$ term in this equation and the energy density is dominated by $\frac{1}{2}m^2\phi^2$, the Friedmann equation implies

$$H \approx \sqrt{\frac{4\pi}{3}} \frac{m}{M_{\text{pl}}} \phi , \quad (17)$$

and the classical motion of the field obeys

$$\dot{\phi} \approx \frac{mM_{\text{pl}}}{\sqrt{12\pi}} . \quad (18)$$

However, superimposed on this classical motion are quantum fluctuations in the field, as shown in Figure 8. On Hubble-radius scales these fluctuations are given by the Gibbons-

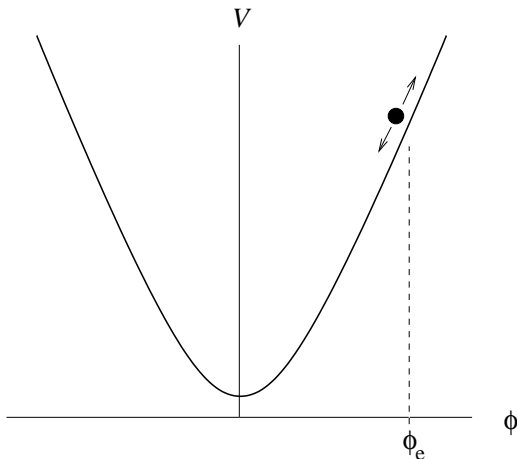


Figure 8: Quantum fluctuations up a potential drive eternal inflation.

Hawking temperature (15),

$$\delta\phi = \frac{H}{2\pi} . \quad (19)$$

These quantum fluctuations become comparable to the classical motion when

$$\delta\phi \approx \frac{\dot{\phi}}{H} , \quad (20)$$

or for any field value $\phi \geq \phi_e$, where

$$\phi_e^2 = \sqrt{\frac{3}{16\pi} \frac{M_{\text{pl}}^3}{m}} . \quad (21)$$

For $m \sim 10^{13}$ GeV, this implies $\phi_e \sim 10^3 M_{\text{pl}}$, with a corresponding Hubble parameter

$$H_e \sim \sqrt{m M_{\text{pl}}} . \quad (22)$$

[Note that these are the values for which inflation is eternal; the values (10) and (11) are the less-stringent requirements for ordinary inflation.] If this point is reached, inflation never truly ends. After each e -fold of expansion, the field is likely to roll *up* the potential in some region of space, while continuing to roll down in most of the comoving volume. However, in the region where the field rolls up, the increased energy density leads to a faster expansion rate and consequently a greater increase in the volume; it's straightforward to show that the physical volume of inflating space continues to grow indefinitely. Although the field along any given geodesic is likely to fall down the potential and reheat as usual, inflation continues to occur somewhere else in the universe.

5.2 Fluctuating into Inflation

The idea we explore in this section is derived directly from the philosophy of eternal inflation, but we begin by imagining that the field is in its vacuum state at the bottom of a potential in a

de Sitter spacetime. Given the nonzero Gibbons-Hawking temperature, there will necessarily be fluctuations in the scalar field over and above the zero-point fluctuations expected in a Minkowski background. There is therefore some chance that the field can spontaneously fluctuate all the way up to ϕ_e in a patch of radius H_e^{-1} , thereby providing an appropriate initial condition for the onset of inflation.³ The probability is of course small, but if we have infinitely long to wait it will eventually occur.

On scales much smaller than the de Sitter radius, spacetime looks flat and we can use finite temperature field theory in flat space to obtain an order of magnitude estimate for the probability that eternal inflation will start. Suppose that at some point in spacetime the inflaton is in its ground state, with a Gaussian wavefunction. The probability density of finding the field at some value $\phi(x)$ is the wavefunction squared:

$$\mathcal{P}[\phi(x), T] = N_{\mathcal{P}} \exp\left(-\frac{\phi^2(x)}{2\langle\phi^2(x)\rangle}\right), \quad (23)$$

where the normalization is

$$N_{\mathcal{P}} = \frac{1}{\sqrt{2\pi\langle\phi^2(x)\rangle}}. \quad (24)$$

The variance $\langle\phi^2(x)\rangle$ is obtained by evaluating the finite temperature correlation function [74]

$$\langle\phi(x), \phi(x')\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} \left(e^{-ik(x-x')} (1 + f(E_k)) + e^{ik(x-x')} f(E_k) \right) \quad (25)$$

at the same point.

However, we need to first average the fluctuations in the field over some scale with a smoothing function f to properly study fluctuations on that scale. We therefore define the smoothed field operator,

$$\bar{\phi}(\mathbf{x}) = \int \phi(\mathbf{y}) g(\mathbf{x} - \mathbf{y}) d^3y. \quad (26)$$

The appropriate scale here is the inflationary Hubble scale. As a smearing function, we will use a Gaussian of width H_e^{-1} ,

$$g(\mathbf{x}) = N_g e^{-\mathbf{x}^2/2H_e^{-2}}, \quad (27)$$

where the appropriate normalization factor is

$$N_g = \left(\frac{H_e}{\sqrt{2\pi}}\right)^3. \quad (28)$$

The answer we obtain will thus correspond to the probability of the field jumping up the potential over one proto-inflationary Hubble volume (much smaller than the Hubble volume of the ambient de Sitter space in which the fluctuation occurs).

³From the point of view of a hypothetical observer outside the fluctuation, the process of spontaneous inflation presumably looks like the creation of a small black hole [29]. Furthermore, we are assuming that the Hubble parameter in the fluctuating region is positive; half of the time it will be negative, leading to a rapid collapse to a singularity inside a black hole.

Averaging the fields in the correlator (25) at equal times, we get

$$\langle \bar{\phi}(\mathbf{x}), \bar{\phi}(\mathbf{x}') \rangle = N_g^2 \int d^3y d^3y' \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} e^{-(\mathbf{y}-\mathbf{x})^2 H_e^2/2} e^{-(\mathbf{y}'-\mathbf{x}')^2 H_e^2/2} \left(e^{i\mathbf{k}\cdot(\mathbf{y}-\mathbf{y}')} (1+f) + e^{-i\mathbf{k}\cdot(\mathbf{y}-\mathbf{y}')} f \right). \quad (29)$$

Integrating over \mathbf{y} and \mathbf{y}' and setting $\mathbf{x} = \mathbf{x}'$, we obtain

$$\langle \bar{\phi}^2(x) \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} e^{-3k^2/H_e^2} (1+2f(E_k)). \quad (30)$$

For bosons, the phase space density is the Bose-Einstein distribution,

$$f(E_k) = \frac{1}{e^{E_k/T_{\text{dS}}} - 1}. \quad (31)$$

The f -independent piece of the correlation function (30) is the usual vacuum-fluctuation divergence present in Minkowski spacetime at zero temperature; we will renormalize this to zero under the assumption that Minkowski spacetime should be stable. The remaining temperature-dependent contribution is a finite integral,

$$\langle \bar{\phi}^2(x) \rangle_{\text{ren}} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k} e^{-3k^2/H_e^2} f(E_k). \quad (32)$$

By changing variables from $|\mathbf{k}|$ to E , we get

$$\langle \bar{\phi}^2(x) \rangle_{\text{ren}} = \frac{1}{2\pi^2} e^{3m^2/H_e^2} \int_m^\infty dE \sqrt{E^2 - m^2} e^{-3E^2/H_e^2} \frac{1}{e^{E_k/T_{\text{dS}}} - 1}, \quad (33)$$

where m is the mass of the scalar, which we will take to be $\sim 10^{13}$ GeV. Given that $\rho_\Lambda \sim 10^{-47}$ GeV⁴, T_{dS} is

$$T_{\text{dS}} = \frac{H_{\text{dS}}}{2\pi} = \sqrt{\frac{2G}{3\pi} \rho_\Lambda} \sim 10^{-43} \text{ GeV}. \quad (34)$$

Because $m \gg T$ (we will henceforth drop the subscript on T), and since E is always larger than m , we can safely assume $e^{E_k/T} \gg 1$. Changing variables again to $y \equiv E/m$, we have

$$\langle \bar{\phi}^2(x) \rangle_{\text{ren}} = \frac{m^2}{2\pi^2} e^{3m^2/H_e^2} \int_1^\infty dy \sqrt{y^2 - 1} e^{-my/T} e^{-3m^2 y^2/H_e^2}. \quad (35)$$

This has no closed form solution. However, the factor $e^{-3m^2 y^2/H_e^2}$ is close to unity over the interval where the other factors are non-negligible, so that it can be safely set to unity. The resulting integral,

$$\langle \bar{\phi}^2(x) \rangle_{\text{ren}} = \frac{m^2}{2\pi^2} e^{3m^2/H_e^2} \int_1^\infty dy \sqrt{y^2 - 1} e^{-my/T}, \quad (36)$$

can be evaluated in terms of a modified Bessel function, K_1 , as

$$\langle \bar{\phi}^2(x) \rangle_{\text{ren}} = \frac{mT}{2\pi^2} K_1(m/T) e^{3m^2/H_e^2}. \quad (37)$$

For large m/T ($\sim 10^{56}$), K_1 is approximately

$$K_1(m/T) \sim \sqrt{\frac{\pi T}{2m}} e^{-m/T}. \quad (38)$$

Finally we find the variance to be

$$\langle \bar{\phi}^2(x) \rangle_{\text{ren}} = \frac{T}{2\pi} \sqrt{\frac{mT}{2\pi}} e^{-m/T} e^{3m^2/H_e^2}. \quad (39)$$

Eternal inflation begins whenever ϕ fluctuates to values larger than ϕ_e (or less than $-\phi_e$). Therefore, the probability per spacetime volume H_e^{-4} that the scalar field spontaneously fluctuates sufficiently far up its potential to induce eternal inflation is

$$P(\phi_e) = 2 \int_{\phi_e}^{\infty} \mathcal{P}[\phi, T] d\phi. \quad (40)$$

Putting in (23), we obtain

$$P(\phi_e) = 1 - \text{erf} \left(\frac{\phi_e}{\sqrt{2 \langle \bar{\phi}^2 \rangle_{\text{ren}}}} \right). \quad (41)$$

The error function $\text{erf}(x)$ can be approximated for large x by

$$\text{erf}(x) \approx 1 - \frac{e^{-x^2}}{x\sqrt{\pi}}. \quad (42)$$

This results in

$$P(\phi_e) = \frac{\exp \left(-\frac{\phi_e^2(x)}{2 \langle \bar{\phi}^2(x) \rangle_{\text{ren}}} \right)}{\sqrt{\pi} \frac{\phi_e}{\sqrt{2 \langle \bar{\phi}^2 \rangle_{\text{ren}}}}}. \quad (43)$$

Substituting (21) and (39) into (43), we find that the probability for the spontaneous onset of eternal inflation in a spacetime volume of radius H_e^{-1} in an ambient de Sitter spacetime with temperature T to be

$$P = \frac{\exp \left(-\pi \frac{M_{\text{pl}}^3}{mT} \sqrt{\frac{6}{mT}} e^{m/T} e^{-3m/M_{\text{pl}}} \right)}{\pi \sqrt{\frac{M_{\text{pl}}^3}{mT}} \sqrt{\frac{6}{mT}} e^{m/T} e^{-3m/M_{\text{pl}}}}. \quad (44)$$

Evaluating this mess for $m \sim 10^{13}$ GeV, $M_{\text{pl}} \sim 10^{19}$ GeV, and $T \sim 10^{-43}$ GeV, we obtain

$$P \sim 10^{-10^{10^{56}}}, \quad (45)$$

an appropriately tiny answer.⁴

The important feature of this probability, calculated in the context of a specific model, is not its actual numerical value, but simply the fact that it is nonzero (which is certainly not a surprise, given our assumptions). The context in which we have performed the calculation – calculating renormalized fluctuations of a scalar field in a fixed de Sitter background, and then imagining that gravitational back-reaction leads to the onset of inflation – is by no means well-understood, although no less so than most discussions of eternal inflation. The crucial point is that it is quite natural for de Sitter to be unstable to the onset of inflation, as has also been argued elsewhere [34, 35]. (The instability studied here is of course different from a possible infrared instability of de Sitter space; see e.g. [75].)

5.3 The Ultra-Large-Scale Structure of the Universe

Given that inflation can spontaneously begin, an important remaining question is whether it is more likely than a fluctuation into a conventional Big-Bang universe. As discussed in Section 3, an appropriate Robertson-Walker universe has a much higher entropy than a proto-inflationary patch, so we might expect the inflationary universe to be correspondingly less likely to be spontaneously created. However, the scenario we are describing does not imagine that our early universe is “chosen randomly” in some measure on the space of initial conditions; rather, that it evolves via a fluctuation from a very specific pre-existing state, namely empty de Sitter. From such a starting point, it is easier for a single mode of wavelength H_e^{-1} to fluctuate up its potential than for a large collection of modes to simultaneously fluctuate into a configuration describing a radiation-dominated Robertson-Walker universe.⁵ This claim is no more dramatic than the claim that it is more likely to find molecules of a gas taking up only one medium-sized corner of a box than to find them spread evenly, *if* we specifically look at the box almost immediately after the gas was released from an even smaller region in the corner. Although the entropy of our early inflationary state is extremely low, it is nevertheless larger than that of the tiny comoving volume of de Sitter from which it arose, in perfect accord with our conventional understanding of the Second Law. (It is crucial to this discussion that the Second Law demands the increase of the total entropy rather than the entropy density, and also that it is sensible to discuss the entropy present along surfaces larger than one Hubble radius.)

There is another, perhaps more persuasive, argument that fluctuations into eternal inflation dominate over those into a conventional Big Bang – namely, that the measure on what is more likely should come from observers in the post-fluctuation universe, rather than from counting events in the pre-fluctuation cold de Sitter space. As has been emphasized often in the eternal-inflation literature, once eternal inflation begins it creates an infinite volume of livable universe in the future. Therefore, even if fluctuations into radiation-dominated universes (or anything similar) are more likely than fluctuations into inflation, most observers

⁴We suspect that this may be smallest positive number in the history of physics, but we haven’t done an exhaustive search to check.

⁵It would obviously be useful to examine this claim more quantitatively. It is interesting to conjecture that fluctuations of the sort we consider would favor the creation of configurations with vanishing Weyl tensor, thus providing a dynamical basis for Penrose’s Weyl Curvature Hypothesis [4].

will find themselves to be living in a post-inflationary region just because of the infinite volume factor associated with eternal inflation. In practice, using this volume factor to calculate sensible probabilities is extremely difficult at best [76, 77, 78, 79, 80, 81, 82, 23]; nevertheless, if our only purpose is to compare inflation to non-inflation, it seems legitimate to appeal to the fecundity of eternal inflation in creating livable regions of spacetime.

Although the probability (45) is quite small, fluctuating into inflation is ultimately inevitable, since the total spacetime volume in the cold de Sitter phase is infinite. One might worry that the de Sitter solution is only metastable, as it would be in the string landscape picture [83, 84, 85, 86, 87, 88, 89, 90, 91, 92] or other theories where the de Sitter phase is a false vacuum liable to decay. However, we don't need the decay rate to be strictly zero, or even smaller than the inflationary-fluctuation rate (45); all we require is that the decay rate per Hubble volume be substantially less than unity. In that case, just as in old inflation, the de Sitter vacuum will never disappear, as the phase transition never percolates; in fact, the physical volume of spacetime in the de Sitter phase will continue to increase, just as in eternal inflation. The total spacetime volume of the de Sitter phase is therefore infinite, and the transition into our proto-inflationary universe is guaranteed eventually to occur. Indeed, it will eventually occur an infinite number of times.

We therefore have a picture in which the universe starts in some generic state defined on a Cauchy surface, and then is allowed to evolve. Local inhomogeneities may collapse to form black holes, which eventually evaporate. The entropy of the configuration is increased by spreading out fluctuations into an ever-larger spatial volume, leaving us with an empty de Sitter solution with a small cosmological constant. The de Sitter phase may or may not be unstable to decay into a state of lower vacuum energy, but the decay rate is assumed to be sufficiently slow that the physical volume of de Sitter grows without bound. Eventually, thermal fluctuations in this background allow a scalar inflaton field to bounce sufficiently high up its potential that eternal inflation begins with a large vacuum energy. Different parts of this inflating region fall down the potential, reheating and evolving into galaxies as in the conventional picture; elsewhere, inflation continues forever.

An interesting feature of this story is that, given the specification of the conditions on the “initial” surface, the same set of events will naturally occur to the *past* as well as to the future. Nothing about our description involved intrinsically time-asymmetric physics, other than the fact that the initial condition was not an equilibrium state with maximal entropy. According to our scenario, this is not in any way a restriction; there is no such thing as a state of maximal entropy, since the entropy can always increase without bound.

We therefore have a picture of the ultra-large-scale structure of the universe, as portrayed in Figure 9. Given some generic state defined on a Cauchy surface, we evolve it to both the past and future, and in both cases it will empty out into a cold de Sitter phase, after which inflation will occasionally begin.⁶ This picture is statistically time-symmetric on very large

⁶A well-known feature of eternal inflation is that it does not escape the problem of singularities, as it cannot be eternal to the past [93]. This is not really an issue for our model. Singularities (or whatever quantum-gravitational phenomenon replaces them in the real world) occur all the time at the center of black holes, and eventually disappear as the black hole evaporates. The singularities demanded by the Borde-Guth-Vilenkin theorems need not be spacelike boundaries for the entire spacetime. Formally, there are no geodesics in our spacetime along which inflation continues at all times to the past and the future.

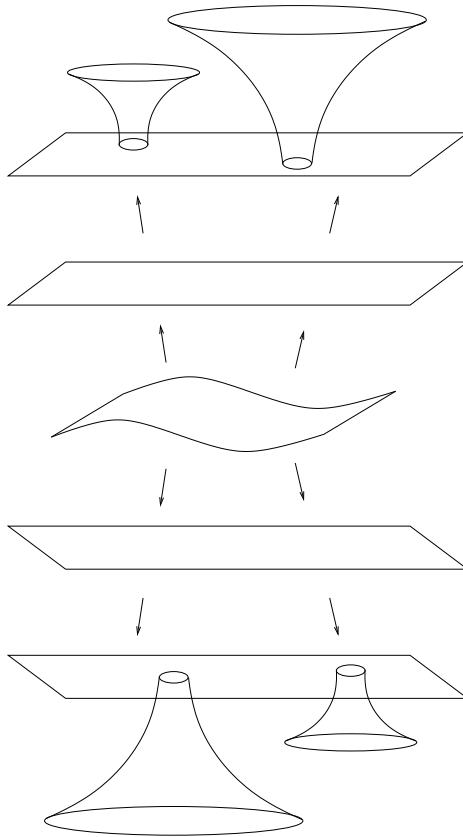


Figure 9: The ultra-large-scale structure of the universe. Starting from a generic state, it can be evolved both forward and backward in time, as it approaches an empty de Sitter configuration. Eventually, fluctuations lead to the onset of inflation in the far past and far future of the starting slice. The arrow of time is reversed in these two regimes.

scales, and satisfies the requirements discussed in Section 2 for a legitimate explanation of our observed arrow of time. Observers in the very far past of our universe will also detect an arrow of time, but one that will be reversed from ours with respect to some (completely unobservable) global time coordinate throughout the entire spacetime. Both sets of observers will think of the others as living in their “past.”

6 Discussion

In this paper we have addressed the question of why the entropy of our observable universe appears to have been extremely small in the past. It has long been suspected that inflation might have something to do with the answer, although the fantastically low entropy of the proto-inflationary universe was a significant obstacle to constructing a convincing picture. The answer to this conundrum must lie in the process by which inflation begins; we have proposed a scenario in which this happens naturally from the evolution of some arbitrarily-

chosen state. It is interesting to note that the recently discovered cosmological constant plays a crucial role in our scenario for generating the arrow of time.

The basic ingredients of our picture are as follows. We consider an arbitrary state of the universe, specified on some Cauchy surface. We argued in Section 4 that the generic evolution of a system coupled to gravity is to dilute excitations via the expansion of spacetime. Black holes and other inhomogeneities may form, but they will eventually decay away, so that the universe approaches empty space. However, in the presence of a positive vacuum energy and an appropriate inflaton field, the resulting de Sitter phase is unstable to the spontaneous onset of inflation, instigated by the thermal fluctuations of the inflaton. If the inflaton fluctuates sufficiently high that eternal inflation can begin, it will continue forever, and new pocket universes will be brought into being in those places where the field rolls down the potential and reheats. This chain of events happens both to the past and the future of the specified Cauchy surface, leading to a statistically time-symmetric universe as portrayed in Figure 9. An arrow of time is dynamically generated in both the past and the future, as the universe continually acts to increase its entropy.

In addition to the desire to understand the origin of the arrow of time, a primary motivation for our study has been to understand the onset of inflation. As discussed in Section 3, the unitarity critique argues that a proto-inflationary patch of spacetime is much lower entropy than that of an ordinary radiation-dominated universe, and hence is less likely to arise as a random fluctuation. In our picture, the answer to this conundrum lies in the fact that the beginning of our observable Big Bang cosmology does not arise as a random choice in a large phase space of initial conditions, but rather comes via a quantum fluctuation from a very specific prior state – an empty de Sitter universe that is the natural consequence of evolution from generic initial conditions.

By taking seriously the ability of spacetime to expand and dilute degrees of freedom, we claim to have shown how an arrow of time can naturally arise dynamically in the course of the evolution from a generic boundary condition. In the classification introduced in Section 2, our proposal imagines that there do not exist any maximum-entropy equilibrium states, but rather that the entropy can increase from any starting configuration. This is not, of course, sufficient; it is also necessary to imagine that the path to increasing the entropy naturally creates regions of spacetime resembling our observable universe. In the presence of a nonzero vacuum energy and an appropriate inflaton field, we suggest that thermal fluctuations from de Sitter space into eternal inflation provide precisely the correct mechanism.

A number of other cosmological scenarios have been proposed in which the Big Bang is not a boundary to spacetime, but simply a phase through which the universe passes. These include the pre-Big-Bang scenario [94, 95], the ekpyrotic and cyclic universe scenarios [96, 97, 98], the Aguirre-Gratton scenario of eternal inflation [99], and Bojowald’s loop-quantum-gravity cosmology [100, 101]. To the best of our understanding, each of these proposals invokes special low-entropy conditions on some Cauchy surface, either asymptotically in the far past or at some moment of minimum size for the universe. In our picture, on the other hand, there is a slice of spacetime on which the entropy is minimized, but that entropy can be arbitrarily large. The Big Bang in our past is not a unique moment in the history of the universe; it is simply one of the many times that inflation spontaneously began from a background de Sitter phase, similar to the proposal of Garriga and Vilenkin [35]. Along

with the fractal distribution of pocket universes to the far past and far future, this feature is another reminder of the importance of overcoming the Robertson-Walker intuition we naturally develop by thinking about the patch of the universe we are actually able to observe.

There are a number of points in our scenario that have yet to be perfectly understood. One obvious point is how the “initial” state is chosen. We have argued that the ultimate evolution to the past and future is essentially insensitive to the details of this state, but it is nevertheless interesting to ask what it may have been. On a more technical level, the idea of spontaneous onset of inflation (and related ideas within the paradigm of eternal inflation) deserves closer investigation. We have calculated the renormalized fluctuations of a scalar field in a fixed de Sitter background, and imagined that the back-reaction of the metric would lead to inflation if the fluctuation were sufficiently large. A more rigorous calculation would have to involve quantum gravity from the start, at least at a semiclassical level. Finally, the issues concerning the number and evolution of degrees of freedom clearly warrant further research. Our assumption has been that the number of degrees of freedom is infinite but fixed, not increasing during inflation. We therefore require a literally infinite number of degrees of freedom to be put in their ground states during the evolution toward de Sitter, before the onset of inflation. It is important that we understand this issue more fully, although doing so may require a deep knowledge of quantum gravity.

An interesting implication of our scenario is the non-privileged role played by the Big Bang of our observable universe. As in other models of eternal inflation, the future history of spacetime takes on a fractal structure of ever-increasing volume; in addition we suggest that a similar structure is found in a time-reversed sense in the far past. The boundary conditions of the universe on ultra-large scales are unlikely to resemble simple Robertson-Walker geometries in any way. Indeed, the cacophony of matter and radiation in our observable patch of universe appears in this picture as a mere byproduct of the relentless evolution of the larger spacetime toward states of higher entropy. Stars and galaxies are seen as exaptations – structures that have found a use other than that for which they were originally developed by evolution.

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