

Prospects of Inflation*

Andrei Linde

Department of Physics, Stanford University, Stanford, CA 94305, USA

E-mail: alinde@stanford.edu

ABSTRACT: I will discuss the development of inflationary theory and its present status, including recent progress in describing de Sitter space and inflationary universe in string theory.

*Extended version of the talk at the Nobel Symposium “Cosmology & String Theory,” August 2003

Contents

1. Introduction	1
2. Brief history of inflation	2
3. Chaotic Inflation	3
4. Hybrid inflation	5
5. Quantum fluctuations and density perturbations	6
6. Eternal inflation	7
7. Inflation and observations	9
8. Alternatives to inflation?	10
9. Shift symmetry and chaotic inflation in supergravity	12
10. Towards Inflation in String Theory	15
10.1 de Sitter vacua in string theory	15
10.2 Inflation in string theory and shift symmetry	15
11. Eternal inflation and stringy landscape	17

1. Introduction

After more than 20 years of its existence, inflationary theory gradually becomes the standard cosmological paradigm. However, we still do not know which of the many versions of inflationary cosmology will be favored by the future observational data. Moreover, it may be quite nontrivial to obtain a natural realization of inflationary theory in the context of the ever changing theory of all fundamental interactions.

Development of inflationary cosmology occurs in many different ways. The original tendency was to concentrate on the basic principles of inflationary theory, such as the

problem of initial conditions, the possibility of eternal inflation, etc. Recent progress in observational cosmology shifted attention towards experimental verification of various inflationary theories. For example, one can parametrize inflationary theories by several slow-roll parameters and find the observational constraints on these parameters. Another approach is to look for inflationary models which can work in the context of the simplest phenomenological theories of elementary particles. A more ambitious trend is to implement inflationary cosmology in string theory. Because of this multitude of goals and methods, different people sometimes have rather different ideas about inflationary theory and its future prospects. In this paper we will try to analyze the situation.

2. Brief history of inflation

The first model of inflationary type was proposed by Alexei Starobinsky [1]. It was based on investigation of conformal anomaly in quantum gravity. This model did not suffer from the graceful exit problem, but it was rather complicated and did not aim on solving homogeneity, horizon and monopole problems.

A much simpler inflationary model with a very clear physical motivation was proposed by Alan Guth [2]. His model, which is now called “old inflation,” was based on the theory of supercooling during the cosmological phase transitions [3]. Even though this scenario did not work, it played a profound role in the development of inflationary cosmology since it contained a very clear explanation how inflation may solve the major cosmological problems.

According to this scenario, inflation is an exponential expansion of the universe in a supercooled false vacuum state. False vacuum is a metastable state without any fields or particles but with large energy density. Imagine a universe filled with such “heavy nothing.” When the universe expands, empty space remains empty, so its energy density does not change. The universe with a constant energy density expands exponentially, thus we have inflation in the false vacuum. Then the false vacuum decays, the bubbles of the new phase collide, and our universe becomes hot.

Unfortunately, if the bubbles of the new phase are formed near each other, their collisions make the universe extremely inhomogeneous. If they are formed far away from each other, each of them represents a separate open universe with a vanishingly small Ω . Both options are unacceptable [2].

This problem was resolved with the invention of the new inflationary theory [5]. In this theory, inflation may begin either in the false vacuum, or in an unstable state at the top of the effective potential. Then the inflaton field ϕ slowly rolls down to the minimum of its effective potential. The motion of the field away from the false vacuum is of crucial importance: density perturbations produced during the slow-roll inflation are inversely proportional to $\dot{\phi}$ [6, 7, 8]. Thus the key difference between the new inflationary scenario and the old one is that the useful part of inflation in the new scenario, which is responsible for the homogeneity of our universe, does *not* occur in the false vacuum state, where $\dot{\phi} = 0$.

This scenario was so popular in the beginning of the 80's that even now most textbooks on astrophysics describe inflation as an exponential expansion during high temperature phase transitions in grand unified theories. Unfortunately, the new inflation scenario was plagued by its own problems. It works only if the effective potential of the field ϕ has a very a flat plateau near $\phi = 0$, which is somewhat artificial. In most versions of this scenario the inflaton field has an extremely small coupling constant, so it could not be in a thermal equilibrium with other matter fields. The theory of cosmological phase transitions, which was the basis for old and new inflation, did not work in such a situation. Moreover, thermal equilibrium requires many particles interacting with each other. This means that new inflation could explain why our universe was so large only if it was very large and contained many particles from the very beginning. Finally, inflation in this theory begins very late. During the preceding epoch the universe can easily collapse or become so inhomogeneous that inflation may never happen [4]. Because of all of these difficulties, no realistic versions of the new inflationary universe scenario have been proposed so far.

Old and new inflation represented a substantial but incomplete modification of the big bang theory. It was still assumed that the universe was in a state of thermal equilibrium from the very beginning, that it was relatively homogeneous and large enough to survive until the beginning of inflation, and that the stage of inflation was just an intermediate stage of the evolution of the universe. In the beginning of the 80's these assumptions seemed most natural and practically unavoidable. On the basis of all available observations (CMB, abundance of light elements) everybody believed that the universe was created in a hot big bang. That is why it was so difficult to overcome a certain psychological barrier and abandon all of these assumptions. This was done with the invention of the chaotic inflation scenario [9]. This scenario resolved all problems of old and new inflation. According to this scenario, inflation may occur even in the theories with simplest potentials such as $V(\phi) \sim \phi^n$. Inflation may begin even if there was no thermal equilibrium in the early universe, and it may start even at the Planckian density, in which case the problem of initial conditions for inflation can be easily resolved [4].

3. Chaotic Inflation

Consider the simplest model of a scalar field ϕ with a mass m and with the potential energy density $V(\phi) = \frac{m^2}{2}\phi^2$. Since this function has a minimum at $\phi = 0$, one may expect that the scalar field ϕ should oscillate near this minimum. This is indeed the case if the universe does not expand, in which case equation of motion for the scalar field coincides with equation for harmonic oscillator, $\ddot{\phi} = -m^2\phi$.

However, because of the expansion of the universe with Hubble constant $H = \dot{a}/a$, an additional term $3H\dot{\phi}$ appears in the harmonic oscillator equation:

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi . \tag{3.1}$$

The term $3H\dot{\phi}$ can be interpreted as a friction term. The Einstein equation for a homoge-

neous universe containing scalar field ϕ looks as follows:

$$H^2 + \frac{k}{a^2} = \frac{1}{6} \left(\dot{\phi}^2 + m^2 \phi^2 \right) . \quad (3.2)$$

Here $k = -1, 0, 1$ for an open, flat or closed universe respectively. We work in units $M_p^{-2} = 8\pi G = 1$.

If the scalar field ϕ initially was large, the Hubble parameter H was large too, according to the second equation. This means that the friction term $3H\dot{\phi}$ was very large, and therefore the scalar field was moving very slowly, as a ball in a viscous liquid. Therefore at this stage the energy density of the scalar field, unlike the density of ordinary matter, remained almost constant, and expansion of the universe continued with a much greater speed than in the old cosmological theory. Due to the rapid growth of the scale of the universe and a slow motion of the field ϕ , soon after the beginning of this regime one has $\ddot{\phi} \ll 3H\dot{\phi}$, $H^2 \gg \frac{k}{a^2}$, $\dot{\phi}^2 \ll m^2\phi^2$, so the system of equations can be simplified:

$$H = \frac{\dot{a}}{a} = \frac{m\phi}{\sqrt{6}} , \quad \dot{\phi} = -m \sqrt{\frac{2}{3}} . \quad (3.3)$$

The first equation shows that if the field ϕ changes slowly, the size of the universe in this regime grows approximately as e^{Ht} , where $H = \frac{m\phi}{\sqrt{6}}$. This is the stage of inflation, which ends when the field ϕ becomes much smaller than $M_p = 1$. Solution of these equations shows that after a long stage of inflation the universe initially filled with the field $\phi = \phi_0 \gg 1$ grows exponentially [4], $a = a_0 e^{\phi_0^2/4}$.

This is as simple as it could be. Inflation does not require supercooling and tunnelling from the false vacuum [2], or rolling from an artificially flat top of the effective potential [5]. It appears in the theories that can be as simple as a theory of a harmonic oscillator [9]. Only after I realized it, I started to believe that inflation is not a trick necessary to fix problems of the old big bang theory, but a generic cosmological regime.

In realistic versions of inflationary theory the duration of inflation could be as short as 10^{-35} seconds. When inflation ends, the scalar field ϕ begins to oscillate near the minimum of $V(\phi)$. As any rapidly oscillating classical field, it loses its energy by creating pairs of elementary particles. These particles interact with each other and come to a state of thermal equilibrium with some temperature T [10, 11, 12]. From this time on, the universe can be described by the usual big bang theory.

The main difference between inflationary theory and the old cosmology becomes clear when one calculates the size of a typical inflationary domain at the end of inflation. Investigation of this question shows that even if the initial size of inflationary universe was as small as the Planck size $l_P \sim 10^{-33}$ cm, after 10^{-35} seconds of inflation the universe acquires a huge size of $l \sim 10^{10^{12}}$ cm! This number is model-dependent, but in all realistic models the size of the universe after inflation appears to be many orders of magnitude greater than the size of the part of the universe which we can see now, $l \sim 10^{28}$ cm. This immediately solves most of the problems of the old cosmological theory [9, 4].

Our universe is almost exactly homogeneous on large scale because all inhomogeneities were exponentially stretched during inflation. The density of primordial monopoles and other undesirable “defects” becomes exponentially diluted by inflation. The universe becomes enormously large. Even if it was a closed universe of a size $\sim 10^{-33}$ cm, after inflation the distance between its “South” and “North” poles becomes many orders of magnitude greater than 10^{28} cm. We see only a tiny part of the huge cosmic balloon. That is why nobody has ever seen how parallel lines cross. That is why the universe looks so flat.

If our universe initially consisted of many domains with chaotically distributed scalar field ϕ (or if one considers different universes with different values of the field), then domains in which the scalar field was too small never inflated. The main contribution to the total volume of the universe will be given by those domains which originally contained large scalar field ϕ . Inflation of such domains creates huge homogeneous islands out of initial chaos. (That is why I called this scenario “chaotic inflation.”) Each homogeneous domain in this scenario is much greater than the size of the observable part of the universe.

The first models of chaotic inflation were based on the theories with polynomial potentials, such as $V(\phi) = \pm \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$. But the main idea of this scenario is quite generic. One should consider any particular potential $V(\phi)$, polynomial or not, with or without spontaneous symmetry breaking, and study all possible initial conditions without assuming that the universe was in a state of thermal equilibrium, and that the field ϕ was in the minimum of its effective potential from the very beginning.

This scenario strongly deviated from the standard lore of the hot big bang theory and was psychologically difficult to accept. Therefore during the first few years after invention of chaotic inflation many authors claimed that the idea of chaotic initial conditions is unnatural, and made attempts to realize the new inflation scenario based on the theory of high-temperature phase transitions, despite numerous problems associated with it. Some authors believed that the theory must satisfy so-called ‘thermal constraints’ which were necessary to ensure that the minimum of the effective potential at large T should be at $\phi = 0$ [13], even though the scalar field in the models they considered was not in a state of thermal equilibrium with other particles. It took several years until it finally became clear that the idea of chaotic initial conditions is most general, and it is much easier to construct a consistent cosmological theory without making unnecessary assumptions about thermal equilibrium and high temperature phase transitions in the early universe.

A short note of terminology: Chaotic inflation occurs in all models with sufficiently flat potentials, including the potentials originally used in new inflation [14]. The versions of chaotic inflation with such potentials for simplicity are often called ‘new inflation’, even though inflation begins there not as in the original new inflation scenario (i.e. not due to the phase transitions with supercooling), but as in the chaotic inflation scenario.

4. Hybrid inflation

In the previous section we considered the simplest chaotic inflation theory based on the

theory of a single scalar field ϕ . The models of chaotic inflation based on the theory of two scalar fields may have some qualitatively new features. One of the most interesting models of this kind is the hybrid inflation scenario [15]. The simplest version of this scenario is based on chaotic inflation in the theory of two scalar fields with the effective potential

$$V(\sigma, \phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2. \quad (4.1)$$

The effective mass squared of the field σ is equal to $-M^2 + g^2\phi^2$. Therefore for $\phi > \phi_c = M/g$ the only minimum of the effective potential $V(\sigma, \phi)$ is at $\sigma = 0$. The curvature of the effective potential in the σ -direction is much greater than in the ϕ -direction. Thus at the first stages of expansion of the universe the field σ rolled down to $\sigma = 0$, whereas the field ϕ could remain large for a much longer time.

At the moment when the inflaton field ϕ becomes smaller than $\phi_c = M/g$, the phase transition with the symmetry breaking occurs. If $m^2\phi_c^2 = m^2M^2/g^2 \ll M^4/\lambda$, the Hubble constant at the time of the phase transition is given by $H^2 = \frac{M^4}{12\lambda}$ (in units $M=1$). If one assumes that $M^2 \gg \frac{\lambda m^2}{g^2}$ and that $m^2 \ll H^2$, then the universe at $\phi > \phi_c$ undergoes a stage of inflation, which abruptly ends at $\phi = \phi_c$.

One of the advantages of this scenario is the possibility to obtain small density perturbations even if coupling constants are large, $\lambda, g = O(1)$. This scenario works if the effective potential has a relatively flat ϕ -direction. But flat directions often appear in supersymmetric theories. This makes hybrid inflation an attractive playground for those who wants to achieve inflation in supergravity and string theory. We will return to this question later.

5. Quantum fluctuations and density perturbations

The vacuum structure in the exponentially expanding universe is much more complicated than in ordinary Minkowski space. The wavelengths of all vacuum fluctuations of the scalar field ϕ grow exponentially during inflation. When the wavelength of any particular fluctuation becomes greater than H^{-1} , this fluctuation stops oscillating, and its amplitude freezes at some nonzero value $\delta\phi(x)$ because of the large friction term $3H\dot{\phi}$ in the equation of motion of the field ϕ . The amplitude of this fluctuation then remains almost unchanged for a very long time, whereas its wavelength grows exponentially. Therefore, the appearance of such a frozen fluctuation is equivalent to the appearance of a classical field $\delta\phi(x)$ that does not vanish after averaging over macroscopic intervals of space and time.

Because the vacuum contains fluctuations of all wavelengths, inflation leads to the creation of more and more new perturbations of the classical field with wavelengths greater than H^{-1} . The average amplitude of such perturbations generated during a typical time interval H^{-1} is given by [16, 17]

$$|\delta\phi(x)| \approx \frac{H}{2\pi}. \quad (5.1)$$

These fluctuations lead to density perturbations that later produce galaxies. The theory of this effect is very complicated [6, 7], and it was fully understood only in the second part of the 80's [8]. The main idea can be described as follows:

Fluctuations of the field ϕ lead to a local delay of the time of the end of inflation, $\delta t = \frac{\delta\phi}{\dot{\phi}} \sim \frac{H}{2\pi\dot{\phi}}$. Once the usual post-inflationary stage begins, the density of the universe starts to decrease as $\rho = 3H^2$, where $H \sim t^{-1}$. Therefore a local delay of expansion leads to a local density increase δ_H such that $\delta_H \sim \delta\rho/\rho \sim \delta t/t$. Combining these estimates together yields the famous result [6, 7, 8]

$$\delta_H \sim \frac{\delta\rho}{\rho} \sim \frac{H^2}{2\pi\dot{\phi}}. \quad (5.2)$$

This derivation is oversimplified; it does not tell, in particular, whether H should be calculated during inflation or after it. This issue was not very important for new inflation where H was nearly constant, but it is of crucial importance for chaotic inflation.

The result of a more detailed investigation [8] shows that H and $\dot{\phi}$ should be calculated during inflation, at different times for perturbations with different momenta k . For each of these perturbations the value of H should be taken at the time when the wavelength of the perturbation becomes of the order of H^{-1} . However, the field ϕ during inflation changes very slowly, so the quantity $\frac{H^2}{2\pi\dot{\phi}}$ remains almost constant over exponentially large range of wavelengths. This means that the spectrum of perturbations of metric is flat.

A detailed calculation in our simplest chaotic inflation model the amplitude of perturbations gives

$$\delta_H \sim \frac{m\phi^2}{5\pi\sqrt{6}}. \quad (5.3)$$

The perturbations on scale of the horizon were produced at $\phi_H \sim 15$ [4]. This, together with COBE normalization $\delta_H \sim 2 \times 10^{-5}$ gives $m \sim 3 \times 10^{-6}$, in Planck units, which is approximately equivalent to 7×10^{12} GeV. Exact numbers depend on ϕ_H , which in its turn depends slightly on the subsequent thermal history of the universe.

The magnitude of density perturbations δ_H in our model depends on the scale l only logarithmically. Since the observations provide us with an information about a rather limited range of l , it is possible to parametrize the scale dependence of density perturbations by a simple power law, $\delta_H \sim l^{(1-n)/2}$. An exactly flat spectrum would correspond to $n = 1$.

Flatness of the spectrum of density perturbations, together with flatness of the universe ($\Omega = 1$), constitute the two most robust predictions of inflationary cosmology. It is possible to construct models where δ_H changes in a very peculiar way, and it is also possible to construct theories where $\Omega \neq 1$, but it is difficult to do so.

6. Eternal inflation

A significant step in the development of inflationary theory was the discovery of the process of self-reproduction of inflationary universe. This process was known to exist in old infla-

tionary theory [2] and in the new one [18], but its significance was fully realized only after the discovery of the regime of eternal inflation in the simplest versions of the chaotic inflation scenario [19, 20]. It appears that in many models large quantum fluctuations produced during inflation which may locally increase the value of the energy density in some parts of the universe. These regions expand at a greater rate than their parent domains, and quantum fluctuations inside them lead to production of new inflationary domains which expand even faster. This leads to an eternal process of self-reproduction of the universe.

To understand the mechanism of self-reproduction one should remember that the processes separated by distances l greater than H^{-1} proceed independently of one another. This is so because during exponential expansion the distance between any two objects separated by more than H^{-1} is growing with a speed exceeding the speed of light. As a result, an observer in the inflationary universe can see only the processes occurring inside the horizon of the radius H^{-1} . An important consequence of this general result is that the process of inflation in any spatial domain of radius H^{-1} occurs independently of any events outside it. In this sense any inflationary domain of initial radius exceeding H^{-1} can be considered as a separate mini-universe.

To investigate the behavior of such a mini-universe, with an account taken of quantum fluctuations, let us consider an inflationary domain of initial radius H^{-1} containing sufficiently homogeneous field with initial value $\phi \gg M_p$. Equation (3.3) implies that during a typical time interval $\Delta t = H^{-1}$ the field inside this domain will be reduced by $\Delta\phi = \frac{2}{\phi}$. By comparison this expression with $|\delta\phi(x)| \approx \frac{H}{2\pi} = \frac{m\phi}{2\pi\sqrt{6}}$ one can easily see that if ϕ is much less than $\phi^* \sim \frac{5}{\sqrt{m}}$, then the decrease of the field ϕ due to its classical motion is much greater than the average amplitude of the quantum fluctuations $\delta\phi$ generated during the same time. But for $\phi \gg \phi^*$ one has $\delta\phi(x) \gg \Delta\phi$. Because the typical wavelength of the fluctuations $\delta\phi(x)$ generated during the time is H^{-1} , the whole domain after $\Delta t = H^{-1}$ effectively becomes divided into $e^3 \sim 20$ separate domains (mini-universes) of radius H^{-1} , each containing almost homogeneous field $\phi - \Delta\phi + \delta\phi$. In almost a half of these domains the field ϕ grows by $|\delta\phi(x)| - \Delta\phi \approx |\delta\phi(x)| = H/2\pi$, rather than decreases. This means that the total volume of the universe containing *growing* field ϕ increases 10 times. During the next time interval $\Delta t = H^{-1}$ the situation repeats. Thus, after the two time intervals H^{-1} the total volume of the universe containing the growing scalar field increases 100 times, etc. The universe enters eternal process of self-reproduction.

Realistic models of elementary particles involve many kinds of scalar fields. For example, in the unified theories of weak, strong and electromagnetic interactions, at least two other scalar fields exist. The potential energy of these scalar fields may have several different minima. This means that the same theory may have different vacuum states, corresponding to different types of symmetry breaking between fundamental interactions, and, as a result, to different laws of low-energy physics.

During the process of eternal inflation in the simplest versions of the chaotic inflation scenario (but not in the new inflation), some parts of the universe spend indefinitely long time at the nearly Planckian density, expanding with the Hubble constant $H = O(1)$, in

Planck mass units. In this regime, all scalar fields persistently experience quantum jumps of the magnitude comparable to the Planck mass. This forces the fields to browse between all possible vacuum states. As a result, the universe becomes divided into infinitely many exponentially large domains that have different laws of low-energy physics [19, 20]. This result may have especially interesting implications in the context of string theory, which allows exponentially large number of different vacuum states [4, 21, 22], see Sect. 11.

7. Inflation and observations

Inflation is not just an interesting theory that can resolve many difficult problems of the standard Big Bang cosmology. This theory made several predictions which can be tested by cosmological observations. Here are the most important predictions:

- 1) The universe must be flat. In most models $\Omega_{total} = 1 \pm 10^{-4}$.
- 2) Perturbations of metric produced during inflation are adiabatic.
- 3) Inflationary perturbations have flat spectrum. In most inflationary models the spectral index $n = 1 \pm 0.2$ ($n = 1$ means totally flat.)
- 4) These perturbations are gaussian.
- 5) Perturbations of metric could be scalar, vector or tensor. Inflation mostly produces scalar perturbations, but it also produces tensor perturbations with nearly flat spectrum, and it does *not* produce vector perturbations. There are certain relations between the properties of scalar and tensor perturbations produced by inflation.
- 6) Inflationary perturbations produce specific peaks in the spectrum of CMB radiation. (For a simple pedagogical interpretation of this effect see e.g. [23]; a detailed theoretical description can be found in [24].)

It is possible to violate each of these predictions if one makes this theory sufficiently complicated. For example, it is possible to produce vector perturbations of metric in the models where cosmic strings are produced at the end of inflation, which is the case in some versions of hybrid inflation. It is possible to have an open or closed inflationary universe, or even a small periodic inflationary universe, it is possible to have models with nongaussian isocurvature fluctuations with a non-flat spectrum. However, it is very difficult to do so, and most of the inflationary models satisfy the simple rules given above.

It is not easy to test all of these predictions. The major breakthrough in this direction was achieved due to the recent measurements of the CMB anisotropy. The latest results based on the WMAP experiment, in combination with the Sloan Digital Sky Survey, are consistent with predictions of the simplest inflationary models with adiabatic gaussian perturbations, with $\Omega = 1.01 \pm 0.02$, and $n = 0.98 \pm 0.03$ [25, 26].

There are still some question marks to be examined, such as the unexpectedly small anisotropy of CMB at large angles [25]. It is not quite clear whether we deal with a real anomaly here or with a manifestation of cosmic variance [27], but in any case, it is quite

significant that all proposed resolutions of this problem are based on inflationary cosmology, see e.g. [28].

8. Alternatives to inflation?

Inflationary scenario is very versatile, and now, after 20 years of persistent attempts of many physicists to propose an alternative to inflation, we still do not know any other way to construct a consistent cosmological theory. Indeed, in order to compete with inflation a new theory should make similar predictions and should offer an alternative solution to many difficult cosmological problems. Let us look at these problems before starting a discussion.

1) Homogeneity problem. Before even starting investigation of density perturbations and structure formation, one should explain why the universe is nearly homogeneous on the horizon scale.

2) Isotropy problem. We need to understand why all directions in the universe are similar to each other, why there is no overall rotation of the universe. etc.

3) Horizon problem. This one is closely related to the homogeneity problem. If different parts of the universe have not been in a causal contact when the universe was born, why do they look so similar?

4) Flatness problem. Why $\Omega \approx 1$? Why parallel lines do not intersect?

5) Total entropy problem. The total entropy of the observable part of the universe is greater than 10^{87} . Where did this huge number come from? Note that the lifetime of a closed universe filled with hot gas with total entropy S is $S^{2/3} \times 10^{-43}$ seconds [4]. Thus S must be huge. Why?

6) Total mass problem. The total mass of the observable part of the universe has mass $\sim 10^{60} M_p$. Note also that the lifetime of a closed universe filled with nonrelativistic particles of total mass M is $\frac{M}{M_P} \times 10^{-43}$ seconds. Thus M must be huge. But why?

7) Structure formation problem. If we manage to explain the homogeneity of the universe, how can we explain the origin of inhomogeneities required for the large scale structure formation?

8) Monopole problem, gravitino problem, etc.

This list is very long. That is why it was not easy to propose any alternative to inflation even before we learned that $\Omega \approx 1$, $n \approx 1$, and that the perturbations responsible for galaxy formation are mostly adiabatic, in agreement with the predictions of the simplest inflationary models.

Despite this difficulty, there was always a tendency to announce that we have eventually found a good alternative to inflation. This was the ideology of the models of structure formation due to topological defects, or textures, which were often described as competitors to inflation, see e.g. [29]. However, it was clear from the very beginning that these theories

at best could solve only one problem (structure formation) out of 8 problems mentioned above. Therefore the true question was not whether one can replace inflation by the theory of cosmic strings/textures, but whether inflation with cosmic strings/textures is better than inflation without cosmic strings/textures. Recent observational data favor the simplest version of inflationary theory, without topological defects, or with an extremely small (few percent) admixture of the effects due to cosmic strings.

A similar situation emerged recently with the introduction of the ekpyrotic scenario [30]. In the original version of this theory it was claimed that this scenario can solve all cosmological problems without using the stage of inflation, i.e. without a prolonged stage of an accelerated expansion of the universe, which was called in [30] “superluminal expansion.” However, this original idea did not work [31, 32], and the idea to avoid “superluminal expansion” was abandoned by the authors of [30]. A more recent version of this scenario, the cyclic scenario [33], uses an infinite number of periods of “superluminal expansion” (i.e. inflation) in order to solve the major cosmological problems. The main difference between the usual inflation and the cyclic inflation, just as in the case of topological defects and textures, is the mechanism of generation of density perturbations. However, since the theory of density perturbations in cyclic inflation requires a solution of the cosmological singularity problem [34, 35], it is difficult to say anything definite about it.

Thus at the moment it is hard to see any real alternative to inflationary cosmology; instead of a competition between inflation and other ideas, we witness a competition between many different models of inflationary theory.

This competition goes in several different directions. First of all, we must try to implement inflation in realistic theories of fundamental interactions. But what do we mean by ‘realistic?’ In the absence of a direct confirmation of M/string theory and supergravity by high energy physics experiments (which may change when we start receiving data from the LHC), the definition of what is realistic becomes increasingly dependent on cosmology and the results of the cosmological observations. In particular, one may argue that those versions of the theory of all fundamental interactions that cannot describe inflation and the present stage of acceleration of the universe are disfavored by observations.

On the other hand, not every theory which can lead to inflation does it in an equally good way. Many inflationary models have been already ruled out by observations. This happened long ago with such models as extended inflation [36] and the simplest versions of “natural inflation” [37]. Recent data from WMAP and SDSS almost ruled out a particular version of chaotic inflation with $V(\phi) \sim \phi^4$ [25, 26].

However, observations test only the last stages of inflation. In particular, they do not say anything about the properties of the inflaton potential at $V(\phi) \gtrsim 10^{-10} M_p^4$. Thus there may exist many different models which describe all observational data equally well. In order to compare such models, one should not only compare their predictions with the results of the cosmological observations, but also carefully examine whether they really solve the main cosmological problems.

One of the most important issues here is the maximal value of energy density during inflation. For example, the simplest chaotic inflation scenario may begin in the universe of the largest possible energy density, of a smallest possible size (Planck length), with the smallest possible mass $M \sim M_p$ and with the smallest possible entropy $S = O(1)$. This provides a true solution to the flatness, horizon, homogeneity, mass and entropy problems. Meanwhile, in the new inflation scenario, inflation occurs on the mass scale 3 orders of magnitude below M_p , when the total size of the universe must be very large. As a result, the initial mass of the universe at the beginning of new inflation must be greater than $10^6 M_p$, and its total entropy must be greater than 10^9 . In other words, in order to explain why the entropy of the universe is greater than 10^{87} one should assume that it was extremely large from the very beginning. This does not look like a real solution of the entropy problem. A similar problem exists in many of the models recently described in [38]. Finally, in cyclic inflation, the process of exponential expansion of the universe occurs only if the total mass of the universe is greater than its present mass $M \sim 10^{60} M_p$ and its total entropy is greater than 10^{87} . This scenario does not solve the flatness, mass and entropy problems.

One can argue [20] that eternal inflation may alleviate some of the problems of the low-scale inflation. Note, however, that eternal inflation, which naturally occurs in the simplest versions of chaotic inflation and in new inflation, does not exist in many popular versions of hybrid inflation. Of course, models of low-scale non-eternal inflation are still much better than the models with no inflation at all, but I do not think that we should settle for the second-best. It would be much better to have a stage of eternal chaotic inflation at nearly Planckian density, which, if needed, may be followed by a stage of low-scale inflation.

Keeping eternal inflation at high density as our ultimate goal, let discuss a possibility to obtain inflation in supergravity and string theory.

9. Shift symmetry and chaotic inflation in supergravity

Most of the existing inflationary models are based on the idea of chaotic initial conditions, which is the trademark of the chaotic inflation scenario. In the simplest versions of chaotic inflation scenario with the potentials $V \sim \phi^n$, the process of inflation occurs at $\phi > 1$, in Planck units. Meanwhile, there are many other models where inflation may occur at $\phi \ll 1$.

There are several reasons why this difference may be important. First of all, some authors argue that the generic expression for the effective potential can be cast in the form

$$V(\phi) = V_0 + \alpha\phi + \frac{m^2}{2}\phi^2 + \frac{\beta}{3}\phi^3 + \frac{\lambda}{4}\phi^4 + \sum_n \lambda_n \frac{\phi^{4+n}}{M_p^n}, \quad (9.1)$$

and then they assume that generically $\lambda_n = O(1)$, see e.g. Eq. (128) in [42]. If this assumption were correct, one would have little control over the behavior of $V(\phi)$ at $\phi > M_p$.

Here we have written M_p explicitly, to expose the implicit assumption made in [42]. Why do we write M_p in the denominator, instead of $1000M_p$? An intuitive reason is that quantum gravity is non-renormalizable, so one should introduce a cut-off at momenta $k \sim M_p$. This is a reasonable assumption, but it does not imply validity of Eq. (9.1). Indeed, the constant part of the scalar field appears in the gravitational diagrams not directly, but only via its effective potential $V(\phi)$ and the masses of particles interacting with the scalar field ϕ . As a result, the terms induced by quantum gravity effects are suppressed not by factors $\frac{\phi^n}{M_p^n}$, but by factors $\frac{V}{M_p^4}$ and $\frac{m^2(\phi)}{M_p^2}$ [4]. Consequently, quantum gravity corrections to $V(\phi)$ become large not at $\phi > M_p$, as one could infer from (9.1), but only at super-Planckian energy density, or for super-Planckian masses. This justifies our use of the simplest chaotic inflation models.

The simplest way to understand this argument is to consider the case where the potential of the field ϕ is a constant, $V = V_0$. Then the theory has a *shift symmetry*, $\phi \rightarrow \phi + c$. This symmetry is not broken by perturbative quantum gravity corrections, so no such terms as $\sum_n \lambda_n \frac{\phi^{4+n}}{M_p^n}$ are generated. This symmetry may be broken by nonperturbative quantum gravity effects (wormholes? virtual black holes?), but such effects, even if they exist, can be made exponentially small [39].

The idea of shift symmetry appears to be very fruitful in application to inflation; we will return to it many times in this paper. However, in some cases the scalar field ϕ itself may have physical (geometric) meaning, which may constrain the possible values of the fields during inflation. The most important example is given by $N = 1$ supergravity.

The effective potential of the complex scalar field Φ in supergravity is given by the well-known expression (in units $M_p = 1$):

$$V = e^K \left[K_{\Phi\bar{\Phi}}^{-1} |D_{\Phi}W|^2 - 3|W|^2 \right]. \quad (9.2)$$

Here $W(\Phi)$ is the superpotential, Φ denotes the scalar component of the superfield Φ ; $D_{\Phi}W = \frac{\partial W}{\partial \Phi} + \frac{\partial K}{\partial \Phi} W$. The kinetic term of the scalar field is given by $K_{\Phi\bar{\Phi}} \partial_{\mu}\Phi \partial_{\mu}\bar{\Phi}$. The standard textbook choice of the Kähler potential corresponding to the canonically normalized fields Φ and $\bar{\Phi}$ is $K = \Phi\bar{\Phi}$, so that $K_{\Phi\bar{\Phi}} = 1$.

This immediately reveals a problem: At $\Phi > 1$ the potential is extremely steep. It blows up as $e^{|\Phi|^2}$, which makes it very difficult to realize chaotic inflation in supergravity at $\phi \equiv \sqrt{2}|\Phi| > 1$. Moreover, the problem persists even at small ϕ . If, for example, one considers the simplest case when there are many other scalar fields in the theory and the superpotential does not depend on the inflaton field ϕ , then Eq. (9.2) implies that at $\phi \ll 1$ the effective mass of the inflaton field is $m_{\phi}^2 = 3H^2$. This violates the condition $m_{\phi}^2 \ll H^2$ required for successful slow-roll inflation (so-called η -problem).

The major progress in SUGRA inflation during the last decade was achieved in the context of the models of the hybrid inflation type, where inflation may occur at $\phi \ll 1$. Among the best models are the F-term inflation, where different contributions to the effective mass term m_{ϕ}^2 cancel [40], and D-term inflation [41], where the dangerous term e^K does not affect the potential in the inflaton direction. A detailed discussion of various

versions of hybrid inflation in supersymmetric theories can be found in [42]. A recent version of this scenario, P-term inflation, which unifies F-term and D-term models, was proposed in [43].

However, hybrid inflation occurs only on a relatively small energy scale, and many of its versions do not lead to eternal inflation. Therefore it would be nice to obtain inflation in a context of a more general class of supergravity models.

This goal seemed very difficult to achieve; it took almost 20 years to find a natural realization of chaotic inflation model in supergravity. Kawasaki, Yamaguchi and Yanagida suggested to take the Kähler potential

$$K = \frac{1}{2}(\Phi + \bar{\Phi})^2 + X\bar{X} \quad (9.3)$$

of the fields Φ and X , with the superpotential $m\Phi X$ [44].

At the first glance, this Kähler potential may seem somewhat unusual. However, it can be obtained from the standard Kähler potential $K = \Phi\bar{\Phi} + X\bar{X}$ by adding terms $\Phi^2/2 + \bar{\Phi}^2/2$, which do not give any contribution to the kinetic term of the scalar fields $K_{\Phi\bar{\Phi}} \partial_\mu \Phi \partial_\mu \bar{\Phi}$. In other words, the new Kähler potential, just as the old one, leads to canonical kinetic terms for the fields Φ and X , so it is as simple and legitimate as the standard textbook Kähler potential. However, instead of the U(1) symmetry with respect to rotation of the field Φ in the complex plane, the new Kähler potential has a *shift symmetry*; it does not depend on the imaginary part of the field Φ . The shift symmetry is broken only by the superpotential.

This leads to a profound change of the potential (9.2): the dangerous term e^K continues growing exponentially in the direction $(\Phi + \bar{\Phi})$, but it remains constant in the direction $(\Phi - \bar{\Phi})$. Decomposing the complex field Φ into two real scalar fields, $\Phi = \frac{1}{\sqrt{2}}(\eta + i\phi)$, one can find the resulting potential $V(\phi, \eta, X)$ for $\eta, |X| \ll 1$:

$$V = \frac{m^2}{2}\phi^2(1 + \eta^2) + m^2|X|^2. \quad (9.4)$$

This potential has a deep valley, with a minimum at $\eta = X = 0$. Therefore the fields η and X rapidly fall down towards $\eta = X = 0$, after which the potential for the field ϕ becomes $V = \frac{m^2}{2}\phi^2$. This provides a very simple realization of eternal chaotic inflation scenario in supergravity [44]. This model can be extended to include theories with different power-law potentials, or models where inflation begins as in the simplest versions of chaotic inflation scenario, but ends as in new or hybrid inflation, see e.g. [45, 46].

It is amazing that for almost 20 years nothing but inertia was keeping us from using the version of the supergravity which was free from the famous η problem. As we will see shortly, the situation with inflation in string theory is very similar, and may have a similar resolution.

10. Towards Inflation in String Theory

10.1 de Sitter vacua in string theory

For a long time, it seemed rather difficult to obtain inflation in M/string theory. The main problem here was the stability of compactification of internal dimensions. For example, ignoring non-perturbative effects to be discussed below, a typical effective potential of the effective 4d theory obtained by compactification in string theory of type IIB can be represented in the following form:

$$V(\sigma, \rho, \phi) \sim e^{\sqrt{2}\sigma - \sqrt{6}\rho} \tilde{V}(\phi) \tag{10.1}$$

Here σ and ρ are canonically normalized fields representing the dilaton field and the volume of the compactified space; ϕ stays for all other fields.

If σ and ρ were constant, then the potential $\tilde{V}(\phi)$ could drive inflation. However, this does not happen because of the steep exponent $e^{\sqrt{2}\sigma - \sqrt{6}\rho}$, which rapidly pushes the dilaton field σ to $-\infty$, and the volume modulus ρ to $+\infty$. As a result, the radius of compactification becomes infinite; instead of inflating, 4d space decompactifies and becomes 10d.

Thus in order to describe inflation one should first learn how to stabilize the dilaton and the volume modulus. The dilaton stabilization was achieved in [47]. The most difficult problem was to stabilize the volume. The solution of this problem was found in [48] (KKLT construction). It consists of two steps.

First of all, due to a combination of effects related to warped geometry of the compactified space and nonperturbative effects calculated directly in 4d (instead of being obtained by compactification), it was possible to obtain a supersymmetric AdS minimum of the effective potential for ρ . This fixed the volume modulus, but in a state with a negative vacuum energy. Then we added an anti- $D3$ brane with the positive energy $\sim \rho^{-3}$. This addition uplifted the minimum of the potential to the state with a positive vacuum energy.

Instead of adding an anti- $D3$ brane, which explicitly breaks supersymmetry, one can add a $D7$ brane with fluxes. This results in the appearance of a D-term which has a similar dependence on ρ , but leads to spontaneous supersymmetry breaking [49]. In either case, one ends up with a metastable dS state which can decay by tunnelling and formation of bubbles of 10d space with vanishing vacuum energy density. The decay rate is extremely small [48], so for all practical purposes, one obtains an exponentially expanding de Sitter space with the stabilized volume of the internal space.¹

10.2 Inflation in string theory and shift symmetry

During the last few years there were many suggestions how to obtain hybrid inflation in string theory by considering motion of branes in the compactified space, see [51, 52] and references therein. The main problem of all of these models was the absence of stabilization

¹It is also possible to find de Sitter solutions in noncritical string theory [50].

of the compactified space. Once this problem was solved for dS space [48], one could try to revisit these models and develop models of brane inflation compatible with the volume stabilization.

The first idea [53] was to consider a pair of D3 and anti-D3 branes in the warped geometry studied in [48]. The role of the inflaton field could be played by the interbrane separation. A description of this situation in terms of the effective 4d supergravity involved Kähler potential

$$K = -3 \log(\rho + \bar{\rho} - k(\phi, \bar{\phi})), \quad (10.2)$$

where the function $k(\phi, \bar{\phi})$ for the inflaton field ϕ , at small ϕ , was taken in the simplest form $k(\phi, \bar{\phi}) = \phi\bar{\phi}$. If one makes the simplest assumption that the superpotential does not depend on ϕ , then the ϕ dependence of the potential (9.2) comes from the term $e^K = (\rho + \bar{\rho} - \phi\bar{\phi})^{-3}$. Expanding this term near the stabilization point $\rho = \rho_0$, one finds that the inflaton field has a mass $m_\phi^2 = 2H^2$. Just like the similar relation $m_\phi^2 = 3H^2$ in the simplest models of supergravity, this is not what we want for inflation.

One way to solve this problem is to consider ϕ -dependent superpotentials. By doing so, one may fine-tune m_ϕ^2 to be $O(10^{-2})H^2$ in a vicinity of the point where inflation occurs [53]. Whereas fine-tuning is certainly undesirable, in the context of string cosmology it may not be a serious drawback. Indeed, if there exist many realizations of string theory [22], then one might argue that all realizations not leading to inflation can be discarded, because they do not describe a universe in which we could live. Meanwhile, those non-generic realizations, which lead to eternal inflation, describe inflationary universes with an indefinitely large and ever-growing volume of inflationary domains. This makes the issue of fine-tuning less problematic.²

Can we avoid fine-tuning altogether? One of the possible ideas is to find theories with some kind of shift symmetry. Another possibility is to construct something like D-term inflation, where the flatness of the potential is not spoiled by the term e^K . Both of these ideas were explored in a recent paper [57] based on the model of D3/D7 inflation in string theory [58]. In this model the Kähler potential is given by

$$K = -3 \log(\rho + \bar{\rho}) - \frac{1}{2}(\phi - \bar{\phi})^2, \quad (10.3)$$

and superpotential depends only on ρ . The shift symmetry $\phi \rightarrow \phi + c$ in this model is related to the requirement of unbroken supersymmetry of branes in a BPS state.

The effective potential with respect to the field ρ in this model coincides with the KKLT potential [48, 49]. In the direction of the real part of the field ϕ , which can be considered an inflaton, the potential is exactly flat, until one adds other fields which break this flatness due to quantum corrections and produce a potential similar to the potential of D-term inflation [57]. The origin of the shift symmetry of the Kähler potential in certain string models was revealed in a recent paper [59]. It is related to special geometry of

²One should note that decreasing of m_ϕ^2 is not the only way to get inflation; one may reach the same goal by considering theories with non-minimal kinetic terms, see e.g. [54, 55, 56].

extended supergravity. This shift symmetry was also studied in a recent version of D3/D7 inflation proposed in [60].

Shift symmetry may help to obtain inflation in other models as well. For example, one may explore the possibility of using the Kähler potential $K = -3 \log(\rho + \bar{\rho} - \frac{1}{2}(\phi - \bar{\phi})^2)$ instead of the potential used in [53]. The modified Kähler potential does not depend on the real part of the field ϕ , which can be considered an inflaton. Therefore the dangerous term $m_\phi^2 = 2H^2$ vanishes, i.e. the main obstacle to the consistent brane inflation in the model of Ref. [53] disappears! A discussion of the possibility to implement shift symmetry in the model of Ref. [53] can be also found in [61].

However, for a while it still remained unclear whether shift symmetry is just a condition which we want to impose on the theory in order to get inflation, or an unavoidable property of the theory, which remains valid even after the KKLT volume stabilization. The answer to this question was found only very recently, and it appears to be model-dependent. It was shown in [62] that in a certain class of models, including D3/D7 models [58, 57, 59, 60], the shift symmetry of the effective 4d theory is not an assumption but an unambiguous consequence of the underlying mathematical structure of the theory. This may allow us to obtain a natural realization of inflation in string theory.

11. Eternal inflation and stringy landscape

Even though we are still at the very first stages of implementing inflation in string theory, it is very tempting to speculate about possible generic features and consequences of such a construction.

First of all, KKLT construction shows that the vacuum energy after the volume stabilization is a function of many different parameters in the theory. One may wonder how many different choices do we actually have. There were many attempts to investigate this issue. For example, many years ago it was argued [63] that there are infinitely many $AdS_4 \times X_7$ vacua of D=11 supergravity. An early estimate of the total number of different 4d string vacua gave the number 10^{1500} [64]. At present we are more interested in counting different flux vacua [65, 22], where the possible numbers, depending on specific assumptions, may vary in the range from 10^{20} to 10^{1000} . Some of these vacuum states with positive vacuum energy can be stabilized using the KKLT approach. Each of such states will correspond to a metastable vacuum state. It decays within a cosmologically large time, which is, however, smaller than the ‘recurrence time’ $e^{S(\phi)}$, where $S(\phi) = \frac{24\pi^2}{V(\phi)}$ is the entropy of dS space with the vacuum energy density $V(\phi)$ [48].

But this is not the whole story; old inflation does not describe our world. In addition to these metastable vacuum states, there should exist various slow-roll inflationary solutions, where the properties of the system practically do not change during the cosmological time H^{-1} . It might happen that such states, corresponding to flat directions in the string theory landscape, exist not only during inflation in the very early universe, but also at the

present stage of the accelerated expansion of the universe. This would simplify obtaining an anthropic solution of the cosmological constant problem along the lines of [67, 65].

If the slow-roll condition $V'' \ll V$ is satisfied all the way from one dS minimum of the effective potential to another, then one can show, using stochastic approach to inflation, that the probability to find the field ϕ at any of these minima, or at any given point between them, is proportional to $e^{S(\phi)}$. In other words, the relative probability to find the field taking some value ϕ_1 as compared to some other value ϕ_0 , is proportional to $e^{\Delta S} = e^{S(\phi_1) - S(\phi_0)}$ [66, 48]. One may argue, using Euclidean approach, that this simple thermodynamic relation should remain valid for the relative probability to find a given point in any of the metastable dS vacua, even if the trajectory between them does not satisfy the slow-roll condition $m^2 \ll H^2$ [68, 69, 70, 71].

The resulting picture resembles eternal inflation in the old inflation scenario. However, now we have an incredibly large number of false vacuum states, plus some states which may allow slow-roll inflation. Once inflation begins, different parts of the universe start wondering from one of these vacuum states to another, so that the universe becomes divided into indefinitely many regions with all possible laws of low-energy physics corresponding to different 4d vacua of string theory [4].

As we already argued, the best inflationary scenario would describe a slow-roll eternal inflation starting at the maximal possible energy density (minimal dS entropy). It would be almost as good to have a low-energy slow-roll eternal inflation. Under certain conditions, such regimes may exist in string theory [53]. However, whereas any of these regimes would make us happy, we already have something that can make us smile. Multi-level eternal inflation of the old inflation type, which appears in string theory in the context of the KKLT construction, may be very useful being combined with the slow-roll inflation, even if the slow-roll inflation by itself is not eternal. We will give a particular example, which is very similar to the one considered in [72].

Suppose we have two noninteracting scalar fields: field ϕ with the potential of the old inflation type, and field χ with the potential which may lead to a slow-roll inflation. Let us assume that the slow-roll inflation belongs to the worst case scenario discussed in Section 8: it occurs only on low energy scale, and it is not eternal. How can we provide initial conditions for such a low-scale inflation?

Let us assume that the Hubble constant at the stage of old inflation is much greater than the curvature of the potential which drives the slow-roll inflation. (This is a natural assumption, considering huge number of possible dS states, and the presumed smallness of energy scale of the slow-roll inflation.) In this case large inflationary fluctuations of the field χ will be generated during eternal old inflation. These fluctuations will give the field χ different values in different exponentially large parts of the universe. When old inflation ends, there will be many practically homogeneous parts of the universe where the field χ will take values corresponding to good initial conditions for a slow-roll inflation. Then the relative fraction of the volume of such parts will grow exponentially.

Moreover, as it was argued in [20], the probability (per unit time and unit volume) to jump back to the eternally inflating regime is always *finite*, even after the field enters the regime where, naively, one would not expect eternal inflation. Each bubble of a new phase which appears during the decay of the eternally inflating dS space is an open universe of an *infinite* volume. Therefore during the slow-roll inflation there always will be some inflationary domains jumping back to the original dS space, so some kind of stationary equilibrium will always exist between various parts of the inflationary universe.

Thus, the existence of many different dS vacua in string theory leads to the regime of eternal inflation. This regime may help us to solve the problem of initial conditions for the slow-roll inflation even in the models where the slow-roll inflation by itself is not eternal and would occur only on a small energy scale.

I am very grateful to the organizers of the Nobel Symposium “Cosmology & String Theory” for the hospitality. It is a special pleasure to express my gratitude to T. Banks, S. Kachru, R. Kallosh, L. Kofman, V. Mukhanov, E. Silverstein, and L. Susskind for many enlightening discussions. This work was supported by NSF grant PHY-0244728.

References

- [1] A. A. Starobinsky, “Spectrum Of Relict Gravitational Radiation And The Early State Of The Universe,” JETP Lett. **30**, 682 (1979) [Pisma Zh. Eksp. Teor. Fiz. **30**, 719 (1979)]; A. A. Starobinsky, “A New Type Of Isotropic Cosmological Models Without Singularity,” Phys. Lett. B **91**, 99 (1980).
- [2] A. H. Guth, “The Inflationary Universe: A Possible Solution To The Horizon And Flatness Problems,” Phys. Rev. D **23**, 347 (1981).
- [3] D. A. Kirzhnits and A. D. Linde, “Symmetry Behavior In Gauge Theories,” Annals Phys. **101**, 195 (1976).
- [4] A.D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland, 1990).
- [5] A. D. Linde, “A New Inflationary Universe Scenario: A Possible Solution Of The Horizon, Flatness, Homogeneity, Isotropy And Primordial Monopole Problems,” Phys. Lett. B **108**, 389 (1982); A. Albrecht and P. J. Steinhardt, “Cosmology For Grand Unified Theories With Radiatively Induced Symmetry Breaking,” Phys. Rev. Lett. **48**, 1220 (1982).
- [6] V. F. Mukhanov and G. V. Chibisov, “Quantum Fluctuation And ‘Nonsingular’ Universe,” JETP Lett. **33**, 532 (1981) [Pisma Zh. Eksp. Teor. Fiz. **33**, 549 (1981)].
- [7] S. W. Hawking, “The Development Of Irregularities In A Single Bubble Inflationary Universe,” Phys. Lett. B **115**, 295 (1982); A. A. Starobinsky, “Dynamics Of Phase Transition In The New Inflationary Universe Scenario And Generation Of Perturbations,” Phys. Lett. B **117**, 175 (1982); A. H. Guth and S. Y. Pi, “Fluctuations In The New Inflationary Universe,” Phys. Rev. Lett. **49**, 1110 (1982); J. M. Bardeen, P. J. Steinhardt and M. S. Turner, “Spontaneous Creation Of Almost Scale - Free Density Perturbations In An Inflationary Universe,” Phys. Rev. D **28**, 679 (1983).

- [8] V. F. Mukhanov, “Gravitational Instability Of The Universe Filled With A Scalar Field,” JETP Lett. **41**, 493 (1985) [Pisma Zh. Eksp. Teor. Fiz. **41**, 402 (1985)]; V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, “Theory Of Cosmological Perturbations,” Phys. Rept. **215**, 203 (1992).
- [9] A. D. Linde, “Chaotic Inflation,” Phys. Lett. B **129**, 177 (1983).
- [10] A. D. Dolgov and A. D. Linde, “Baryon Asymmetry In Inflationary Universe,” Phys. Lett. B **116**, 329 (1982); L. F. Abbott, E. Farhi and M. B. Wise, “Particle Production In The New Inflationary Cosmology,” Phys. Lett. B **117**, 29 (1982).
- [11] L. Kofman, A. D. Linde and A. A. Starobinsky, “Reheating after inflation,” Phys. Rev. Lett. **73**, 3195 (1994) [arXiv:hep-th/9405187]; L. Kofman, A. D. Linde and A. A. Starobinsky, “Towards the theory of reheating after inflation,” Phys. Rev. D **56**, 3258 (1997) [arXiv:hep-ph/9704452].
- [12] G. N. Felder, J. Garcia-Bellido, P. B. Greene, L. Kofman, A. D. Linde and I. Tkachev, “Dynamics of symmetry breaking and tachyonic preheating,” Phys. Rev. Lett. **87**, 011601 (2001) [arXiv:hep-ph/0012142]; G. N. Felder, L. Kofman and A. D. Linde, “Tachyonic instability and dynamics of spontaneous symmetry breaking,” Phys. Rev. D **64**, 123517 (2001) [arXiv:hep-th/0106179].
- [13] B. A. Ovrut and P. J. Steinhardt, “Supersymmetry And Inflation: A New Approach,” Phys. Lett. B **133**, 161 (1983); B. A. Ovrut and P. J. Steinhardt, “Inflationary Cosmology And The Mass Hierarchy In Locally Supersymmetric Theories,” Phys. Rev. Lett. **53**, 732 (1984); B. A. Ovrut and P. J. Steinhardt, “Locally Supersymmetric Cosmology And The Gauge Hierarchy,” Phys. Rev. D **30**, 2061 (1984); B. A. Ovrut and P. J. Steinhardt, “Supersymmetric Inflation, Baryon Asymmetry And The Gravitino Problem,” Phys. Lett. B **147**, 263 (1984).
- [14] A. D. Linde, “Primordial Inflation Without Primordial Monopoles,” Phys. Lett. B **132** (1983) 317.
- [15] A. D. Linde, “Axions in inflationary cosmology,” Phys. Lett. B **259**, 38 (1991); A. D. Linde, “Hybrid inflation,” Phys. Rev. D **49**, 748 (1994) [astro-ph/9307002].
- [16] A. Vilenkin and L. H. Ford, “Gravitational Effects Upon Cosmological Phase Transitions,” Phys. Rev. D **26**, 1231 (1982).
- [17] A. D. Linde, “Scalar Field Fluctuations In Expanding Universe And The New Inflationary Universe Scenario,” Phys. Lett. B **116**, 335 (1982).
- [18] P. J. Steinhardt, “Natural Inflation,” In: *The Very Early Universe*, ed. G.W. Gibbons, S.W. Hawking and S. Siklos, Cambridge University Press, (1983); A. D. Linde, “Nonsingular Regenerating Inflationary Universe,” Cambridge University preprint Print-82-0554 (1982); A. Vilenkin, “The Birth Of Inflationary Universes,” Phys. Rev. D **27**, 2848 (1983).
- [19] A. D. Linde, “Eternally Existing Selfreproducing Chaotic Inflationary Universe,” Phys. Lett. B **175**, 395 (1986).
- [20] A. D. Linde, D. A. Linde and A. Mezhlumian, “From the Big Bang theory to the theory of a stationary universe,” Phys. Rev. D **49**, 1783 (1994) [arXiv:gr-qc/9306035].
- [21] L. Susskind, “The anthropic landscape of string theory,” arXiv:hep-th/0302219.

- [22] M. R. Douglas, “The statistics of string / M theory vacua,” JHEP **0305**, 046 (2003) [arXiv:hep-th/0303194]; S. Ashok and M. R. Douglas, “Counting flux vacua,” arXiv:hep-th/0307049; M. R. Douglas, “Statistics of String vacua,” arXiv:hep-ph/0401004.
- [23] S. Dodelson, “Coherent phase argument for inflation,” AIP Conf. Proc. **689**, 184 (2003) [arXiv:hep-ph/0309057].
- [24] V. Mukhanov, “CMB-slow, or How to Estimate Cosmological Parameters by Hand,” arXiv:astro-ph/0303072.
- [25] H. V. Peiris *et al.*, “First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Implications for inflation,” Astrophys. J. Suppl. **148**, 213 (2003) [arXiv:astro-ph/0302225].
- [26] M. Tegmark *et al.* [SDSS Collaboration], “Cosmological parameters from SDSS and WMAP,” arXiv:astro-ph/0310723.
- [27] G. Efstathiou, “The Statistical Significance of the Low CMB Multipoles,” arXiv:astro-ph/0306431; G. Efstathiou, “A Maximum Likelihood Analysis of the Low CMB Multipoles from WMAP,” arXiv:astro-ph/0310207.
- [28] C. R. Contaldi, M. Peloso, L. Kofman and A. Linde, “Suppressing the lower Multipoles in the CMB Anisotropies,” JCAP **0307**, 002 (2003) [arXiv:astro-ph/0303636].
- [29] D. Spergel and N. Turok, “Textures and cosmic structure,” Scientific American **266**, 52 (1992).
- [30] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, “The ekpyrotic universe: Colliding branes and the origin of the hot big bang,” Phys. Rev. D **64**, 123522 (2001) [arXiv:hep-th/0103239].
- [31] R. Kallosh, L. Kofman and A. D. Linde, “Pyrotechnic universe,” Phys. Rev. D **64**, 123523 (2001) [arXiv:hep-th/0104073].
- [32] R. Kallosh, L. Kofman, A. D. Linde and A. A. Tseytlin, “BPS branes in cosmology,” Phys. Rev. D **64**, 123524 (2001) [arXiv:hep-th/0106241].
- [33] P. J. Steinhardt and N. Turok, “Cosmic evolution in a cyclic universe,” Phys. Rev. D **65**, 126003 (2002) [arXiv:hep-th/0111098].
- [34] H. Liu, G. Moore and N. Seiberg, “Strings in a time-dependent orbifold,” JHEP **0206**, 045 (2002) [arXiv:hep-th/0204168].
- [35] G. T. Horowitz and J. Polchinski, “Instability of spacelike and null orbifold singularities,” Phys. Rev. D **66**, 103512 (2002) [arXiv:hep-th/0206228].
- [36] D. La and P. J. Steinhardt, “Extended Inflationary Cosmology,” Phys. Rev. Lett. **62**, 376 (1989) [Erratum-ibid. **62**, 1066 (1989)].
- [37] K. Freese, J. A. Frieman and A. V. Olinto, “Natural Inflation With Pseudo - Nambu-Goldstone Bosons,” Phys. Rev. Lett. **65**, 3233 (1990).
- [38] D. H. Lyth, “Which is the best inflation model?,” arXiv:hep-th/0311040.
- [39] R. Kallosh, A. Linde, D. Linde and L. Susskind, “Gravity and global symmetries,” Phys. Rev. **D52**, 912 (1995) [hep-th/9502069].

- [40] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, “False vacuum inflation with Einstein gravity,” *Phys. Rev. D* **49**, 6410 (1994) [astro-ph/9401011]; G. R. Dvali, Q. Shafi and R. Schaefer, “Large scale structure and supersymmetric inflation without fine tuning,” *Phys. Rev. Lett.* **73**, 1886 (1994) [hep-ph/9406319]; A. D. Linde and A. Riotto, “Hybrid inflation in supergravity,” *Phys. Rev. D* **56**, 1841 (1997) [arXiv:hep-ph/9703209].
- [41] P. Binetruy and G. Dvali, “D-term inflation,” *Phys. Lett. B* **388**, 241 (1996) [hep-ph/9606342]; E. Halyo, “Hybrid inflation from supergravity D-terms,” *Phys. Lett. B* **387**, 43 (1996) [hep-ph/9606423].
- [42] D. H. Lyth and A. Riotto, “Particle physics models of inflation and the cosmological density perturbation,” *Phys. Rept.* **314**, 1 (1999) [hep-ph/9807278].
- [43] R. Kallosh and A. Linde, “P-term, D-term and F-term inflation,” *JCAP* **0310**, 008 (2003) [arXiv:hep-th/0306058].
- [44] M. Kawasaki, M. Yamaguchi and T. Yanagida, “Natural chaotic inflation in supergravity,” *Phys. Rev. Lett.* **85**, 3572 (2000) [arXiv:hep-ph/0004243].
- [45] M. Yamaguchi and J. Yokoyama, “New inflation in supergravity with a chaotic initial condition,” *Phys. Rev. D* **63**, 043506 (2001) [arXiv:hep-ph/0007021]; M. Yamaguchi, “Natural double inflation in supergravity,” *Phys. Rev. D* **64**, 063502 (2001) [arXiv:hep-ph/0103045].
- [46] M. Yamaguchi and J. Yokoyama, “Chaotic hybrid new inflation in supergravity with a running spectral arXiv:hep-ph/0307373.
- [47] S. B. Giddings, S. Kachru and J. Polchinski, “Hierarchies from fluxes in string compactifications,” *Phys. Rev. D* **66**, 106006 (2002) [arXiv:hep-th/0105097].
- [48] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, “De Sitter vacua in string theory,” *Phys. Rev. D* **68**, 046005 (2003) [arXiv:hep-th/0301240].
- [49] C. P. Burgess, R. Kallosh and F. Quevedo, “de Sitter string vacua from supersymmetric D-terms,” *JHEP* **0310**, 056 (2003) [arXiv:hep-th/0309187].
- [50] E. Silverstein, “(A)dS backgrounds from asymmetric orientifolds,” arXiv:hep-th/0106209; A. Maloney, E. Silverstein and A. Strominger, “De Sitter space in noncritical string theory,” arXiv:hep-th/0205316.
- [51] G. R. Dvali and S. H. H. Tye, “Brane inflation,” *Phys. Lett. B* **450**, 72 (1999) [arXiv:hep-ph/9812483].
- [52] F. Quevedo, “Lectures on String/Brane Cosmology,” hep-th/0210292.
- [53] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister and S. P. Trivedi, “Towards inflation in string theory,” *JCAP* **0310**, 013 (2003) [arXiv:hep-th/0308055].
- [54] C. Armendariz-Picon, T. Damour and V. Mukhanov, “k-inflation,” *Phys. Lett. B* **458**, 209 (1999) [arXiv:hep-th/9904075].
- [55] S. Dimopoulos and S. Thomas, “Discretuum versus continuum dark energy,” *Phys. Lett. B* **573**, 13 (2003) [arXiv:hep-th/0307004].
- [56] E. Silverstein and D. Tong, “Scalar speed limits and cosmology: Acceleration from D-acceleration,” arXiv:hep-th/0310221.

- [57] J. P. Hsu, R. Kallosh and S. Prokushkin, “On brane inflation with volume stabilization,” *JCAP* **0312**, 009 (2003) [arXiv:hep-th/0311077].
- [58] R. Kallosh, “ $N = 2$ supersymmetry and de Sitter space,” arXiv:hep-th/0109168; C. Herdeiro, S. Hirano and R. Kallosh, “String theory and hybrid inflation/acceleration,” *JHEP* **0112** (2001) 027 [arXiv:hep-th/0110271]; K. Dasgupta, C. Herdeiro, S. Hirano and R. Kallosh, “D3/D7 inflationary model and M-theory,” *Phys. Rev. D* **65**, 126002 (2002) [arXiv:hep-th/0203019].
- [59] C. Angelantonj, R. D’Auria, S. Ferrara and M. Trigiante, “ $K3 \times T^{**2}/Z(2)$ orientifolds with fluxes, open string moduli and critical points,” arXiv:hep-th/0312019.
- [60] F. Koyama, Y. Tachikawa and T. Watari, “Supergravity analysis of hybrid inflation model from D3-D7 system,” arXiv:hep-th/0311191.
- [61] H. Firouzjahi and S. H. H. Tye, “Closer towards inflation in string theory,” arXiv:hep-th/0312020.
- [62] J. P. Hsu and R. Kallosh, “Volume Stabilization and the Origin of the Inflaton Shift Symmetry in String Theory,” hep-th/0402047.
- [63] M. J. Duff, B. E. W. Nilsson and C. N. Pope, “Kaluza-Klein Supergravity,” *Phys. Rept.* **130**, 1 (1986).
- [64] W. Lerche, D. Lust and A. N. Schellekens, “Chiral Four-Dimensional Heterotic Strings From Selfdual Lattices,” *Nucl. Phys. B* **287**, 477 (1987); M. J. Duff, “Not The Standard Superstring Review,” CERN-TH-4749-87 *Proc. of Int. School of Subnuclear Physics, The Super-World II, Erice, Italy, Aug 6-14, 1987*.
- [65] R. Bousso and J. Polchinski, “Quantization of four-form fluxes and dynamical neutralization of the cosmological constant,” *JHEP* **0006**, 006 (2000) [arXiv:hep-th/0004134].
- [66] A. D. Linde, “Quantum creation of an open inflationary universe,” *Phys. Rev. D* **58**, 083514 (1998) [arXiv:gr-qc/9802038].
- [67] A. D. Linde, “The Inflationary Universe,” *Rept. Prog. Phys.* **47**, 925 (1984); A. D. Sakharov, “Cosmological Transitions With A Change In Metric Signature,” *Sov. Phys. JETP* **60**, 214 (1984) [*Zh. Eksp. Teor. Fiz.* **87**, 375 (1984)]; A.D. Linde, “Inflation And Quantum Cosmology,” Print-86-0888 (June 1986) in *300 Years of Gravitation*, ed. by S.W. Hawking and W. Israel, Cambridge University Press, Cambridge (1987); S. Weinberg, “Anthropic Bound On The Cosmological Constant,” *Phys. Rev. Lett.* **59**, 2607 (1987); A. Vilenkin, “Predictions From Quantum Cosmology,” *Phys. Rev. Lett.* **74**, 846 (1995) [arXiv:gr-qc/9406010]; G. Efstathiou, *MNRAS* **274**, L73 (1995); H. Martel, P. R. Shapiro and S. Weinberg, “Likely Values of the Cosmological Constant,” *Astrophys. J.* **492**, 29 (1998) [arXiv:astro-ph/9701099]; J. Garriga and A. Vilenkin, “On likely values of the cosmological constant,” *Phys. Rev. D* **61**, 083502 (2000) [arXiv:astro-ph/9908115]; J. L. Feng, J. March-Russell, S. Sethi and F. Wilczek, “Saltatory relaxation of the cosmological constant,” *Nucl. Phys. B* **602**, 307 (2001) [arXiv:hep-th/0005276].
- [68] S. W. Hawking and I. G. Moss, “Supercooled Phase Transitions In The Very Early Universe,” *Phys. Lett. B* **110**, 35 (1982).
- [69] K. M. Lee and E. J. Weinberg, “Decay Of The True Vacuum In Curved Space-Time,” *Phys. Rev. D* **36**, 1088 (1987).

- [70] J. Garriga and A. Vilenkin, “Recycling universe,” *Phys. Rev. D* **57**, 2230 (1998) [arXiv:astro-ph/9707292].
- [71] L. Dyson, M. Kleban and L. Susskind, “Disturbing implications of a cosmological constant,” *JHEP* **0210**, 011 (2002) [arXiv:hep-th/0208013]; N. Goheer, M. Kleban and L. Susskind, “The trouble with de Sitter space,” *JHEP* **0307**, 056 (2003) [arXiv:hep-th/0212209].
- [72] A. D. Linde, “Chaotic Inflation With Constrained Fields,” *Phys. Lett. B* **202**, 194 (1988).