Cosmological Models with No Big Bang

Wun-Yi Shu (許文郁)
Institute of Statistics
National Tsing Hua University
Hsinchu 30013, Taiwan
E-mail: shu@stat.nthu.edu.tw
Abstract

In the late 1990s, observations of Type Ia supernovae led to the astounding discovery that the universe is expanding at an accelerating rate. The explanation of this anomalous acceleration has been one of the great problems in physics since that discovery. In this article we propose cosmological models that can explain the cosmic acceleration without introducing a cosmological constant into the standard Einstein field equation, negating the necessity for the existence of dark energy. There are four distinguishing features of these models: 1) the speed of light and the gravitational “constant” are not constant, but vary with the evolution of the universe, 2) time has no beginning and no end, 3) the spatial section of the universe is a 3-sphere, and 4) the universe experiences phases of both acceleration and deceleration. One of these models is selected and tested against current cosmological observations of Type Ia supernovae, and is found to fit the redshift-luminosity distance data quite well.
I. INTRODUCTION

In the late 1990s, observations of Type Ia supernovae made by two groups, the Supernova Cosmology Project [1] and the High-z Supernova Search Team [2], indicated that the universe appears to be expanding at an accelerating rate. The current mainstream explanation of the accelerating expansion of the universe is to introduce a mysterious form of energy—the so called dark energy that opposes the self-attraction of matter. Two proposed forms for dark energy are the cosmological constant, which can be viewed physically as the vacuum energy, and scalar fields, sometimes called quintessence, whose cosmic expectation values evolve with time. Currently, in the spatially flat $\Lambda CDM$ model of cosmology, dark energy accounts for nearly three-quarters of the total mass-energy of the universe [3]. The introduction of dark energy raises several theoretical difficulties, and understanding the anomalous cosmic acceleration has become one of the greatest challenges of theoretical physics. There are a number of excellent review papers on this issue [4-6].

In this article we propose cosmological models that can explain the accelerating universe without introducing a cosmological constant into the standard Einstein field equation, negating the necessity for the existence of dark energy. There are four distinguishing features of these models:
The speed of light and the gravitational “constant” are not constant, but vary with the evolution of the universe.

Time has no beginning and no end; i.e., there is neither a big bang nor a big crunch singularity.

The spatial section of the universe is a 3-sphere, ruling out the possibility of a flat or hyperboloid geometry.

The universe experiences phases of both acceleration and deceleration.

One of these models is selected and tested against the current cosmological observations, and is found to fit the redshift-luminosity distance data quite well.

This article has the following structure: In the next section, the cosmological models are developed, with the details of the calculations presented in the Appendix. In Sec. 3, the dynamical evolution of the universe is determined by solving the Einstein field equation under various conditions. In Sec. 4, a selected model is tested against the observations of Type Ia supernovae. Four data sets available in the literature are included in the test. Finally, the results are discussed in Sec. 5.

Throughout this article we follow the sign conventions of Wald [7]. In particular, we use metric signature $-+++$, define the Riemann and the Ricci tensors by equations (3.2.3) and (3.2.25) of Wald [7] respectively, and employ abstract index notation to denote tensors. Greek indices, running from 0 to 3, are used to denote
components of tensors while Latin indices are used to denote tensors. Einstein’s summation convention is assumed.

II. COSMOLOGICAL MODELS

A cosmological model is defined by specifying: 1) the spacetime geometry determined by a metric $g_{ab}$, 2) the mass-energy distributions described in terms of a stress-energy-momentum tensor $T_{ab}$, and 3) the interaction of the geometry and the mass-energy, which is depicted through a field equation.

A. The spacetime metric

Under the assumption that on the large scale the universe is homogeneous and isotropic and expressed in the synchronous time coordinate and co-moving spatially spherical/hyperbolic coordinates $(t, \psi, \theta, \phi)$, the line element of the spacetime metric $g_{ab}$ takes the form [7]

$$
\begin{aligned}
    ds^2 &= -c^2 dt^2 + a^2(t) \left\{ d\psi^2 + \sin^2 \psi \left( d\theta^2 + \sin^2 \theta \ d\phi^2 \right) \\
    &= d\psi^2 + \psi^2 \left( d\theta^2 + \sin^2 \theta \ d\phi^2 \right) \ , \\
    &= d\psi^2 + \sinh^2 \psi \left( d\theta^2 + \sin^2 \theta \ d\phi^2 \right)
\end{aligned}
$$

(2.1)

where $c$ is the speed of light and the three options listed to the right of the left bracket correspond to the three possible spatial geometries: a 3-sphere, 3-dimensional
flat space, and a 3-dimensional hyperboloid, respectively. The metric of form (2.1) is called the Friedmann-Robertson-Walker (FRW) metric.

We view the speed of light as simply a conversion factor between time and space in spacetime. It is simply one of the properties of the spacetime geometry. Since the universe is expanding, we speculate that the conversion factor somehow varies in accordance with the evolution of the universe, hence the speed of light varies with cosmic time. Denoting the speed of light as a function of cosmic time by $c(t)$, we modify the FRW metric as

$$ds^2 = -c^2(t)dt^2 + a^2(t)\left(d\psi^2 + \sin^2\psi\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right).$$  \hspace{1cm} (2.2)

**B. The stress-energy-momentum tensor**

The universe is assumed to contain both matter and radiation. The content of the universe is described in terms of a stress-energy-momentum tensor $T^{ab}$. We shall take $T^{ab}$ to have the general perfect fluid form

$$T^{ab} = \left(\rho + \frac{P}{c^2}\right)u^a u^b + P g^{ab},$$

where $u^a$, $\rho$ and $P$ are, respectively, a time-like vector field representing the 4-velocity, the proper average mass density, and the pressure as measured in the instantaneous rest frame of the cosmological fluid.
C. The field equation

In a cosmology where the speed of light is assumed constant, the interaction between the curvature of spacetime at any event and the matter content at that event is depicted through Einstein’s field equation

\[ G_{ab} \equiv R_{ab} - \frac{1}{2} R \ g_{ab} = 8\pi \frac{G}{c^4} T_{ab}, \tag{2.3} \]

where \( R_{ab} \) is the Ricci tensor, \( R \) is the curvature scalar, and \( G \) is the Newtonian gravitational constant. In a cosmology with a varying \( c \) and varying \( G \), one needs a new field equation for attaining consistency [8]. Noting that \( G/c^2 \) is the conversion factor that translates a unit of mass into a unit of length, we postulate that \( c \) and \( G \) vary in such a way that \( G(t)/c(t)^2 \) must be absolutely constant with respect to the cosmic time \( t \). We can make \( G(t)/c(t)^2 = 1 \) by choosing proper units of mass and length. Accordingly, we speculate that in a cosmology with a varying \( c \) and varying \( G \), the field equation describing the interaction between the spacetime geometry and the distribution of mass-energy is given as

\[ G_{ab} \equiv R_{ab} - \frac{1}{2} R \ g_{ab} = 8\pi \left(1/c^2 \right) T_{ab} \equiv 8\pi \ T^{*\,ab}, \tag{2.4} \]

where \( T^{*\,ab} \equiv \left(1/c^2 \right) T_{ab} \). Due to the Bianchi identity, the contravariant tensor \( T^{*\,ab} \) satisfies the equation of motion

\[ \nabla_a T^{*\,ab} = 0. \]

In particular,
\[ \nabla_{\mu} T^{\mu 0} = 0 , \]

which, after some straightforward algebra (see Appendix, section 1 and 2), yields

\[ \dot{\rho}(t) + 3 \left( \rho(t) + \frac{P(t)}{c^2(t)} \right) \frac{\dot{a}(t)}{a(t)} = 0 . \]

Thus, for a universe composed of pressure-free dust \((P = 0)\) we have

\[ \frac{d}{dt} \left[ \rho(t) a^3(t) \right] = 0, \]

or equivalently,

\[ \rho(t) a^3(t) = \text{constant} , \quad (2.5) \]

while for a universe composed of dust and radiation \((P = \rho c^2 / 3)\) we obtain

\[ \rho(t) a^4(t) = \text{constant} . \quad (2.6) \]

Equations (2.5) and (2.6) express conservation of mass-energy.

III. DYNAMICS OF THE UNIVERSE

In this section we determine the dynamical behavior, which is characterized by the functions \(a(t)\) and \(c(t)\) in metric (2.2), of a universe as described by our cosmological models. To obtain predictions for the dynamical evolution, we substitute metric (2.2) into the field equation (2.4) and solve for \(a(t)\) and \(c(t)\). Computing the components of \(G_{ab}\) in terms of \(a(t)\) and \(c(t)\), and then plugging the expressions
for them into equation (2.4), after some tedious but straightforward calculation (see Appendix, section 1 and 3), we arrive at the evolution equations for a homogeneous and isotropic universe:

\[
\left[ \frac{\ddot{a}(t)}{c(t)} \right]^2 = \frac{8\pi}{3} \rho(t)a^2(t) - k, \text{ and }
\]  

\[
\frac{\dddot{a}(t)}{a(t)} - \frac{\ddot{a}(t)}{a(t)} \frac{\dot{c}(t)}{c(t)} = -\frac{4\pi}{3} \left( \rho(t) + 3\frac{p(t)}{c^2(t)} \right) c^2(t),
\]  

where \( k = 1 \) for the 3-sphere, \( k = 0 \) for flat space, and \( k = -1 \) for the hyperboloid.

Using equation (2.5) or, respectively equation (2.6), we rewrite equation (3.1), for a universe composed of dust only, as

\[
\left[ \frac{\ddot{a}(t)}{c(t)} \right]^2 = 2\frac{M}{a(t)} - k,
\]

where \( M \equiv 4\pi\rho(t)a^3(t)/3 \), which is constant by equation (2.5); while for a universe composed of dust and radiation, as

\[
\left[ \frac{\ddot{a}(t)}{c(t)} \right]^2 = 2\frac{M'}{a^2(t)} - k,
\]

where \( M' \equiv 4\pi\rho(t)a^4(t)/3 \), which is constant by equation (2.6).

There are two unknown functions, \( c(t) \) and \( a(t) \), to be determined. To solve equations (3.2) and (3.3) we need a further postulate on the relationship between \( c(t) \) and \( a(t) \). For this we argue as follows: When converting the magnitude of increment in time, \( dt \), into that in length, Nature needs a universal standard to refer to. Noting
that the concept of time arises from the observation that the distribution of mass-energy contained in the universe is dynamic and the rate of change, $\dot{\rho}(t)$, of the cosmological density is the very quantity that manifests the dynamicity of a homogeneous universe, we postulate that $\dot{\rho}(t)$ is the standard taken by Nature. If the distribution was static, $\dot{\rho}(t) \equiv 0$, the concept of time would have no meaning. The cosmological density plays the role of ultimate clock in a homogeneous universe.

Accordingly, when being converted into that in length, the magnitude of increment in time, $dt$, is normalized with $\dot{\rho}(t)$. The conversion between time and length can then be expressed as

$$dt \mapsto \left( \kappa_0 / |\dot{\rho}(t)| \right) dt,$$

where $\kappa_0$ is constant with respect to the cosmic time $t$. Since the speed of light in a vacuum $c(t)$ is viewed as simply a conversion factor between time and length in spacetime, we also have the conversion

$$dt \mapsto c(t)dt.$$

By comparing the right hand sides of these two conversions, we conclude

$$c(t) \propto \frac{1}{|\dot{\rho}(t)|}.$$

Equation (2.5) gives

$$\dot{\rho}(t) \propto \frac{\dot{a}(t)}{a^4(t)}.$$

Accordingly, we speculate that
\[ c(t) = \kappa \left( \frac{a^4(t)}{|\dot{a}(t)|} \right), \quad (3.4) \]

where \( \kappa \) is constant with respect to the cosmic time \( t \). We are now ready to solve equations (3.2) and (3.3) for \( a(t) \) and \( c(t) \). Given equations (2.5) and (3.4), equation (3.2) is redundant, so equation (3.3) is all we need to arrive at a solution. We will solve equation (3.3) for the universe composed of pressure-free dust and with spatially 3-sphere geometry \((k = 1)\) explicitly and discuss the other cases briefly.

Substituting (3.4) into equation (3.3) yields

\[ \kappa^{-2} \left[ \frac{\dot{a}(t)}{a^2(t)} \right]^4 = \frac{2M}{a(t)} - 1. \quad (3.5) \]

Simplifying and preparing (3.5) for integration results in

\[ \kappa^{-1/2} \dot{a}(t) = \pm \left[ 2M - a(t) \right]^{3/4} a(t)^{7/4} \]

\[ \Rightarrow \kappa^{-1/2} \frac{da}{dt} = \pm \left( 2M - a \right)^{3/4} a^{7/4} \]

\[ \Rightarrow \kappa^{1/2} \frac{dt}{da} = \pm \left( 2M - a \right)^{-1/4} a^{-7/4} \]

\[ \Rightarrow dt = \pm \kappa^{-1/2} \left( 2M - a \right)^{-1/4} a^{-7/4} da. \]

Carrying out the integration leads to

\[ t(a) = \pm \kappa^{-1/2} \int_{2M}^{a} \left( 2M - x \right)^{-1/4} x^{-7/4} dx, \quad 0 < a \leq 2M \]

\[ = \pm \frac{1}{2\kappa^{1/2} M} \int_{0}^{a/2M} (1-u)^{-1/4} u^{-7/4} du, \quad 0 < a \leq 2M \]

\[ = \pm \frac{2}{3\kappa^{1/2} M} \left( \frac{2M}{a} - 1 \right)^{3/4}, \quad 0 < a \leq 2M. \quad (3.6) \]
We have chosen the time origin \((t = 0)\) to be that when \(a\) achieves its maximum value \(2M\). Setting \(\sigma = \frac{2}{3\kappa^{1/2}}M\) and solving equation (3.6) for \(a\) in terms of \(t\), we finally arrive at

\[
a(t) = \frac{2M}{1 + \left(\frac{t}{\sigma}\right)^{4/3}}, \quad -\infty < t < \infty.
\]

(3.7)

In this model, \(a(t)\) is the hyper-radius of the universe at cosmic time \(t\). The radius will get smaller and smaller as \(t\) approaches \(\pm\infty\), however it can never reach zero, and therefore, time has no beginning and no end, and there is neither a big bang nor a big crunch singularity. Setting \(\gamma(t) = a(t)/2M\), the universe is accelerating in the epoch when \(\gamma(t) < 7/8\) and is decelerating when \(\gamma(t) > 7/8\). The graph of \(a(t)/2M\) versus \(t/\sigma\) is displayed in Figure 1.
FIG. 1. The evolution of the universe composed of pressure-free dust and with spatially 3-sphere geometry. The hyper-radius of the universe, $a(t)$, can never reach zero. The universe is accelerating in the epoch when $\gamma < 7/8$ and is decelerating when $\gamma > 7/8$.

From equations (3.4) and (3.7), the speed of light in a vacuum, as a function of cosmic time $t$, can be calculated as

$$c(t) = \frac{\left(8 \frac{M}{3 \sigma}\right)}{\left[1 + (t/\sigma)^{4/3}\right]^{2} \left(|t|/\sigma\right)^{1/3}}$$

$$= \left(4\kappa^{1/2}M^{2}\right)\gamma^{2}(t) \left[\frac{1}{\gamma(t)} - 1\right]^{-1/4}.$$  \hspace{1cm} (3.8)

Since the speed of light $c$, wavelength $\lambda$, and frequency $\nu$ are related by $c = \lambda \nu$, a varying $c$ could be interpreted in different ways. We assume that a varying $c$ arises from a varying $\lambda$ with $\nu$ kept constant. We further assume that the relation
between the energy $E$ of a photon and the wavelength $\lambda$ of its associated electromagnetic wave is given by equation $E(t) = \eta / \lambda(t)$, where $\eta$ is a constant that does not vary over cosmic time. Consequently, from relation $\lambda(t) = c(t)/\nu$, it follows that $E(t) = \left[\eta / c(t)\right] \nu \equiv \hbar(t)\nu$. Therefore, the so called Planck’s constant $\hbar$ actually varies with the evolution of the universe.

Following the same procedure as above, the solutions for the other five cases are given as follows:

- For a universe composed of pressure-free dust and with spatially flat geometry,

$$t(a) / \sigma = (3/4) \int \limits_1^{a/2M} u^{-7/4} du = 1 - \left(\frac{a}{2M}\right)^{-3/4}, \quad 0 < a < \infty,$$

$$a(t) = \frac{2M}{\left(1 - t / \sigma\right)^{4/3}}, \quad -\infty < t < \sigma.$$

We have chosen the time origin $(t = 0)$ to be that when $a$ reaches the value $2M$. In this case $a(t)$ will blow up at a finite future time $t = \sigma$. The graph of $a(t)/2M$ versus $t/\sigma$ is displayed in Figure 2.

- For a universe composed of pressure-free dust and with spatially hyperboloid geometry,
\[
t(a)/\sigma = (3/4) \int_{1}^{a/2M} (1+u)^{-1/4} u^{-7/4} du, \quad 0 < a < \infty.
\]

\[
= 2^{3/4} - \left(1 + \frac{2M}{a}\right)^{3/4}, \quad 0 < a < \infty.
\]

Equation (3.9)

Solving equation (3.9) for \(a\) in terms of \(t\) yields

\[
a(t) = \frac{2M}{\left(2^{3/4} - t/\sigma\right)^{4/3} - 1}, \quad -\infty < t < \sigma \left(2^{3/4} - 1\right).
\]

In this case \(a(0) = 2M\) and \(a(t)\) will blow up at a finite future time \(t = \sigma \left(2^{3/4} - 1\right)\).

The graph of \(a(t)/2M\) versus \(t/\sigma\) is displayed in Figure 2.

**FIG. 2.** The dynamics of two versions of the universe composed of pressure-free dust: with spatially flat geometry, and with spatially hyperboloid geometry. In both universes \(a(t)\) can never reach zero and will blow up at a finite future time.
For a universe composed of dust and radiation, and with spatially 3-sphere geometry,

\[ t(a)/\sigma' = \pm \int_1^{a/\sqrt{2M'}} (1-u^2)^{-1/4} u^{-2} du, \quad 0 < a \leq \sqrt{2M'}, \]

or equivalently

\[ t(a)/\sigma' = \pm (1/2) \int_1^{a^2/2M'} (1-u)^{-1/4} u^{-3/2} du, \quad 0 < a \leq \sqrt{2M'}, \]

where

\[ \sigma' = \frac{1}{(\kappa')^{1/2} (2M')^{3/4}}, \quad \text{and} \quad c(t) = (\kappa') \left( \frac{\dot{a}(t)}{|\ddot{a}(t)|} \right). \]

We have chosen the time origin \((t=0)\) to be that when \(a\) achieves its maximum value \(\sqrt{2M'}\). In this case \(a(t) \sim |t|^{-1}\), as \(t \to \pm \infty\).

For a universe composed of dust and radiation, and with spatially flat geometry,

\[ t(a)/\sigma' = \int_1^{a/\sqrt{2M'}} u^{-2} du = 1 - \frac{\sqrt{2 M'}}{a}, \]

\[ a(t) = \frac{\sqrt{2 M'}}{1 - t/\sigma'}. \]

In this case \(a(0) = \sqrt{2M'}\) and \(a(t)\) will blow up at a finite future time \(t = \sigma'\).

For a universe composed of dust and radiation, and with spatially hyperboloid geometry,

\[ t(a)/\sigma' = \pm \int_1^{a/\sqrt{2M'}} (1+u^2)^{-1/4} u^{-2} du, \quad 0 < a < \infty, \]
or equivalently,
\[ t(a) / \sigma' = \left( \frac{1}{2} \right)^{a^2 / 2M^4} \int_1^{a^2 / 2M^4} (1 + u)^{-1/4} u^{-3/2} du . \]

In this case \( a(0) = \sqrt{2M^4} \) and \( a(t) \) will blow up at a finite future time before \( t = \sigma^4 \).

From these results, we see that a spatially flat or spatially hyperboloid geometry is not feasible to describe our universe, since in each case \( a(t) \) will blow up at a finite future time.

**IV. THE COSMOLOGICAL REDSHIFT AND DATA FITTING**

In this section we test the model for the universe composed of pressure-free dust and with spatially 3-sphere geometry against cosmological observations. Theoretical predictions of luminosity distance as a function of redshift will be compared with cosmological observations of Type Ia supernovae. Four data sets available in the literature are included in the test: 1) 18 supernovae from the Calán/Tololo Supernova Survey [1], 2) 42 Type Ia supernovae discovered by the Supernova Cosmology Project [1], 3) 44 supernovae assembled by Astier et al. (2006) [3], and 4) 73 Type Ia supernovae discovered by the Supernova Legacy Survey [3].
Suppose that a photon of frequency (wavelength) $\nu_e$ ($\lambda_e$) is emitted at cosmic time $t_c$ by an isotropic observer $E$ with fixed spatial coordinates $(\psi_E, \theta_E, \phi_E)$, and further suppose this photon is observed at time $t_o$ by another isotropic observer $O$ at fixed co-moving coordinates. We may take $O$ to be at the origin of our spatial coordinate system. Let $\nu_o$ ($\lambda_o$) be the frequency (wavelength) measured by this second observer. The redshift factor, $z$, is given by

$$1 + z \equiv \frac{\lambda_o}{\lambda_e} = \frac{c(t_o)/\nu_o}{c(t_c)/\nu_e} = \frac{c(t_o) a(t_o)}{c(t_c) a(t_c)}.$$ 

Substituting the expression for $a(t)$ in (3.7) and that for $c(t)$ in (3.8) into the above equation yields

$$1 + z = \left[1 + \left(\frac{t_c}{\sigma}\right)^{4/3}\right]^{3/4} \left(\frac{t_c}{\sigma}\right)^{1/3} \left[1 + \left(\frac{t_o}{\sigma}\right)^{4/3}\right]^{3/4} \left(\frac{t_o}{\sigma}\right)^{1/3}.$$ 

(4.1)

By setting

$$\gamma(t) \equiv a(t)/2M, \quad \gamma_e \equiv \gamma(t_c), \quad \text{and} \quad \gamma_o \equiv \gamma(t_o),$$

equation (4.1) can be simplified as

$$\left(\frac{1}{\gamma_e}\right)^3 \left(\frac{1}{\gamma_e} - 1\right)^{1/4} = (1 + z) \left(\frac{1}{\gamma_o}\right)^3 \left(\frac{1}{\gamma_o} - 1\right)^{1/4}.$$ 

(4.2)

From metric (2.2) and the fact that $ds = d\theta = d\phi = 0$ along the photon path, we have

$$\psi_E = \int_{t_c}^{t_o} \frac{c(t)}{a(t)} dt.$$ 

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Carrying out the integration leads to
\[ \int_{t_e}^{t_o} \frac{c(t)}{a(t)} \, dt = 2 \left[ \tan^{-1} \left( \frac{1}{\gamma_e} - 1 \right)^{1/2} - \tan^{-1} \left( \frac{1}{\gamma_o} - 1 \right)^{1/2} \right]. \] (4.3)

The proper distance between \( E \) and \( O \) at cosmic time \( t_o \) is
\[ d_p (z) = a (t_o) \psi_E, \]
and the luminosity distance between \( E \) and \( O \) at cosmic time \( t_o \) is evaluated as
\[ d_L (z) = a (t_o) (1 + z) \sin \psi_E \]
\[ = 2 M \gamma(t_o) (1 + z) \sin \psi_E. \] (4.4)

In the data sets included in this test, the luminosity distance is given as the stretch-luminosity corrected effective B-band peak magnitude \([1]\), \( m_B^{\text{effective}} \), and the magnitude-redshift relation is expressed as
\[ m_B^{\text{effective}} = M_B + 5 \log d_L(z) + 25, \]
where \( M_B \) is a parameter believed to be constant for all supernovae of Type Ia \([9-11]\). From equations (4.3) and (4.4), our model will predict the theoretical value \( m_{\gamma_o, \beta}(z) \) with unknown parameters \( \gamma_o \) and \( \beta \) as follows:
\[ m_B^{\text{effective}} = \beta + 5 \log \left( \frac{1}{\gamma_o} (1+z) \sin [z, \gamma_o] \right) = m_{\gamma_o, \beta}(z), \]
where \( \beta \equiv M_B + 5 \log 2M + 25 \) and
\[ [z, \gamma_o] = 2 \left[ \tan^{-1} \left( \frac{1}{\gamma_o} (z) - 1 \right)^{1/2} - \tan^{-1} \left( \frac{1}{\gamma_o} - 1 \right)^{1/2} \right]. \] (4.5)

For a given \( \gamma_o \), the quantity \( \gamma_e(z) \) in (4.5), as a function of redshift factor \( z \), is defined implicitly by equation (4.2). The best-fit parameters are determined by minimizing the
quantity

\[ \sum_{i=1}^{n} d^2 \left[ (z_i, m_i), m_{\gamma_0, \beta} (\bullet) \right] \]

over \(0 < \gamma_0 \leq 1\) and \(\beta \geq 0\), i.e.,

\[ (\gamma^*, \beta^*) = \arg \min \sum_{0 < \gamma_0 \leq 1, \beta \geq 0} d^2 \left[ (z_i, m_i), m_{\gamma_0, \beta} (\bullet) \right], \]

where \((z_i, m_i), i = 1, 2, \ldots, n,\) are the observations and

\[ d^2 \left[ (z_i, m_i), m_{\gamma_0, \beta} (\bullet) \right] = \min_{z \geq 0} \left[ \left( \frac{z_i - z}{\sigma_i} \right)^2 + \left( \frac{m_i - m_{\gamma_0, \beta} (z)}{\sigma_{m_i}} \right)^2 \right]. \]

**Figure 3** shows the Hubble diagram of corrected effective rest-frame B magnitude as a function of redshift \(z\) for the 177 supernovae contained in the test data sets. The theoretical predictions, \(m_{\gamma_0, \beta} (z)\), of the model, with \(\gamma_0 = 0.001\) and \(\beta = 49.321\), fit the observations quite well.
FIG. 3. Hubble diagram for: (a) 62 low-redshift Type Ia supernovae, 18 from the Calán/Tololo Supernova Survey (Data Set 1) and 44 assembled by the Supernova Legacy Survey (Data Set 3), and (b) 115 high-redshift Type Ia supernovae, 42 discovered by the Supernova Cosmology Project (Data Set 2) and 73 by the Supernova Legacy Survey (Data Set 4). The solid curve is the theoretical value $m_{\gamma_0, \beta}(z)$ as predicted by our model with parameters $\gamma_0 = 0.001$ and $\beta = 49.321$.

V. DISCUSSION

In the Friedmann cosmology [12], a homogeneous and isotropic universe must have begun in a singular state. Hawking and Penrose [13] proved that singularities are
generic features of cosmological solutions only if general relativity is correct and the universe is filled with as much matter and radiation as we observe. The prediction of singularities represents a breakdown of general relativity. Many authors felt that the idea of singularities was repulsive and spoiled the beauty of Einstein’s theory. There were therefore a number of attempts [14-17] to avoid the conclusion that there had been a big bang, but were all abandoned eventually. Negating the existence of singularities restores beauty to Einstein’s theory of general relativity.

Cosmological constant $\Lambda$ was introduced into the field equation of gravity by Einstein as a modification of his original theory to ensure a static universe. After Hubble’s redshift observations [18] indicated that the universe is not static, the original motivation for the introduction of $\Lambda$ was lost. However, $\Lambda$ has been reintroduced on numerous occasions when it might be needed to reconcile theory and observations, in particular with the discovery of cosmic acceleration in the 1990s. With our models successfully explaining the accelerating universe without the introduction of $\Lambda$, the concept of cosmological constant shall be discarded again from the point of view of logical economy, as suggested by Einstein [19].

Beginning with Dirac [20] in 1937, some physicists have speculated that several so called physical constants may actually vary. Theories for a varying speed of light (VSL) have been proposed independently by Petit [21-23] from 1988, Moffat [24] in
1993, and then Barrow [25] and Albrecht and Magueijo [26] in 1999 as an alternative way to cosmic inflation [27-29] of solving several cosmological puzzles such as the flatness and horizon problems (for a detailed discussion of these problems, see Weinberg [30], section 4.1). For reviews of VSL, see Magueijo [31]. In the standard big bang cosmological models, the flatness problem arises from observation that the initial condition of the density of matter and energy in the universe is required to be fine-tuned to a very specific critical value for a flat universe. With our models asserting that the spatial section of the universe is a 3-sphere, the flatness problem disappears automatically. The horizon problem of the standard cosmology is a consequence of the existence of the big bang origin and the deceleration in the expansion of the universe. Without the big bang origin and with the universe being accelerating in the epoch when $\gamma(t) < 7/8$, our models may thus provide a solution to the horizon problem.

Essentially, this work is a novel theory about how the magnitudes of the three basic physical dimensions, mass, time, and length are converted into each other, or equivalently, a novel theory about how the geometry of spacetime and the distribution of mass-energy interact. The theory resolves problems in cosmology, such as those of the big bang, dark energy, and flatness, in one fell stroke by postulating that

$$c(t) = \kappa \left( \frac{a^4(t)}{\left| \dot{a}(t) \right|} \right) \ \text{and} \ \tau \equiv \frac{G(t)}{c^2(t)} = \text{constant}.$$
Since there are three basic physical dimensions, any cosmological model requires two constants. Einstein took \( c \) and \( G \) as the two constants, whereas we assert that the two constants are \( \kappa \), the factor relating to the conversion between time and length, and \( \tau \), the conversion factor between mass and length. These two constants, \( \kappa \) and \( \tau \), together with \( \eta \), the constant relating the energy of a photon and the wavelength of its associated electromagnetic wave, can be used to define the natural units of measurement for the three basic physical dimensions. Using dimensional analysis, we obtain:

\[
\begin{align*}
\text{• the natural unit of mass} & \equiv \frac{\eta}{\kappa \tau^3} \\
\text{• The natural unit of time} & \equiv \frac{1}{\kappa \tau \eta} \\
\text{• The natural unit of length} & \equiv \frac{\tau \eta}{\kappa}.
\end{align*}
\]

Although, from expression (3.8), the speed of light in a vacuum becomes infinite, hence a singularity, at cosmic time \( t=0 \), it is of interest to note that

\[
\int_{-\infty}^{\infty} c(t) \, dt = 2 \pi M < \infty.
\]

In this sense, we may call the singularity a pseudo singularity.

By comparing cosmological models, we refute the claim [32] that the time variation of a dimensional quantity such as the speed of light has no intrinsic physical
significance. We illustrate our point as follows: In Friedmann’s closed universe, which resulted from the constancy of the speed of light, the time span is a closed interval, from big bang to big crunch, while in ours the time span is an open interval, with neither beginning nor end. The two models can be discriminated by the topological structures of their time spans—the former is compact, whereas the latter is not [33].

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APPENDIX

1. Calculations for the Components of $\Gamma^c_{ab}$ and $R_{ab}$

For the case of 3-sphere geometry, in the synchronous time coordinate and co-moving spatial spherical coordinates $(t, \psi, \theta, \phi)$, the covariant components of metric $g_{ab}$ are

$$
\begin{bmatrix}
-g^2(t) & 0 & 0 & 0 \\
0 & a^2(t) & 0 & 0 \\
0 & 0 & a^2(t)\sin^2\psi & 0 \\
0 & 0 & 0 & a^2(t)\sin^2\psi\sin^2\theta
\end{bmatrix},
$$

and the contravariant components are

$$
\begin{bmatrix}
-g^2(t) & 0 & 0 & 0 \\
0 & a^2(t) & 0 & 0 \\
0 & 0 & a^2(t)\sin^2\psi & 0 \\
0 & 0 & 0 & a^2(t)\sin^2\psi\sin^2\theta
\end{bmatrix}^{-1}.
$$

From equation (3.1.30) of Wald [7],

$$
\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left( \partial^\mu g_{\nu\sigma} + \partial^\nu g_{\mu\sigma} - \partial^\sigma g_{\mu\nu} \right),
$$

after some tedious but straightforward calculation, we find that the only non-vanishing components of the Christoffel symbol $\Gamma^c_{ab}$ are

$$
\Gamma^0_{00} = \frac{\dot{c}(t)}{c(t)}, \quad \Gamma^0_{11} = \frac{a(t)\ddot{a}(t)}{c^2(t)}, \quad \Gamma^0_{22} = \frac{a(t)\ddot{a}(t)\sin^2\psi}{c^2(t)},
$$

$$
\Gamma^0_{33} = \frac{a(t)\ddot{a}(t)\sin^2\psi\sin^2\theta}{c^2(t)},
$$

$$
\Gamma^0_{12} = \Gamma^0_{21} = \frac{a(t)\ddot{a}(t)\sin^2\psi}{c^2(t)}, \quad \Gamma^0_{13} = \Gamma^0_{31} = \frac{a(t)\ddot{a}(t)\sin^2\psi\sin^2\theta}{c^2(t)},
$$

$$
\Gamma^0_{23} = \Gamma^0_{32} = \frac{a(t)\ddot{a}(t)\sin^2\psi\sin^2\theta}{c^2(t)}.
$$
\[
\Gamma^l_{01} = \Gamma^l_{10} = \frac{\dot{a}(t)}{a(t)}, \quad \Gamma^l_{22} = -\sin \psi \cos \psi, \quad \Gamma^l_{33} = -\sin \psi \cos \psi \sin^2 \theta, \\
\Gamma^2_{02} = \Gamma^2_{02} = \frac{\dot{a}(t)}{a(t)}, \quad \Gamma^2_{12} = \Gamma^2_{21} = \frac{\cos \psi}{\sin \psi}, \quad \Gamma^2_{33} = -\sin \theta \cos \theta, \\
\Gamma^3_{03} = \Gamma^3_{03} = \frac{\dot{a}(t)}{a(t)}, \quad \Gamma^3_{13} = \Gamma^3_{31} = \frac{\cos \psi}{\sin \psi}, \quad \Gamma^3_{23} = \Gamma^3_{32} = \frac{\cos \theta}{\sin \theta}, \\
\]

where the dots denote derivatives with respect to \( t \). Substituting these expressions for the components of \( \Gamma^c_{ab} \) into those for the components of the Ricci tensor \( R_{ab} \),

\[
R_{\mu\nu} = \partial_\beta \Gamma^\beta_{\mu\nu} - \partial_\mu \Gamma^\beta_{\beta\nu} + \Gamma^\alpha_{\mu\nu} \Gamma^\beta_{\alpha\beta} - \Gamma^\alpha_{\beta\nu} \Gamma^\beta_{\alpha\mu},
\]

gives

\[
R_{00} = 3 \frac{\ddot{a}(t) c(t)}{a(t) c(t)} - 3 \frac{\dddot{a}(t)}{a(t)}, \\
R_{11} = 2 \left[ \frac{\ddot{a}(t)}{c(t)} \right]^2 + \frac{a(t) \dddot{a}(t)}{c^2(t)} - \frac{a(t) \dddot{a}(t) c(t)}{c^3(t)} + 2, \\
R_{22} = R_{11} \sin^2 \psi, \quad R_{33} = R_{11} \sin^2 \psi \sin^2 \theta, \text{ and}
\]

\[
R_{\mu\nu} = 0, \text{ for } \mu \neq \nu.
\]

From \( R_{\mu\nu} = R_{\mu\lambda} G^{\lambda\nu} \), it follows that

\[
R^0_0 = 3 \left[ \frac{\dddot{a}(t)}{a(t)} - \frac{\dddot{a}(t) c(t)}{a(t) c(t)} \right] \frac{1}{c^2(t)}, \text{ and}
\]

\[
R^1_1 = R^2_2 = R^3_3 = 2 \left[ \frac{\dddot{a}(t)}{a(t) c(t)} \right]^2 + \frac{\dddot{a}(t)}{a(t) c^2(t)} - \frac{\dddot{a}(t) c(t)}{a(t) c^3(t)} + \frac{2}{a^2(t)}. 
\]
Hence

\[ R = R_{\mu}^{\mu} = \frac{6}{c^2(t)} \left[ \dddot{a}(t) + \frac{\dot{a}(t)^2}{a(t)} - \frac{\dot{a}(t) \ddot{c}(t)}{a(t) \dot{c}(t)} + \frac{c^2(t)}{\dot{a}(t)} \right]. \]

2. Equations of motion

Expressed in the synchronous time coordinate and co-moving spatial spherical/hyperbolic coordinates \((t, \psi, \theta, \phi) = [x^\mu]\), the 4-velocity of the fluid is simply \([u^\mu] = [\partial x^\mu / \partial t] = (1, 0, 0, 0)\). Hence the contravariant components of \(T_{\text{ab}} = \left( \rho + \frac{P}{c^2} \right) u^a u^b + g_{\text{ab}}\) are

\[
\begin{bmatrix}
\rho(t) & 0 & 0 & 0 \\
0 & P(t) a^{-2}(t) & 0 & 0 \\
0 & 0 & P(t) a^{-2}(t) \sin^{-2} \psi & 0 \\
0 & 0 & 0 & P(t) a^{-2}(t) \sin^{-2} \psi \sin^{-2} \theta
\end{bmatrix},
\]

and the covariant components are

\[
\begin{bmatrix}
\rho(t) c^4(t) & 0 & 0 & 0 \\
0 & P(t) a^2(t) & 0 & 0 \\
0 & 0 & P(t) a^2(t) \sin^2 \psi & 0 \\
0 & 0 & 0 & P(t) a^2(t) \sin^2 \psi \sin^2 \theta
\end{bmatrix}.
\]

Set \(T^*_{\text{ab}} = c^{-2}(t) T_{\text{ab}}\) and \(T^*_{\text{ab}} = c^{-2}(t) T_{\text{ab}}\). From the contravariant version of the field equation \(G_{\text{ab}} = 8\pi T^*_{\text{ab}}\) and the Bianchi identity, we see that \(T^*_{\text{ab}}\) satisfies the...
equation of motion

\[ \nabla_a T^{*ab} = 0 \ . \]

In particular,

\[ \nabla \mu T^{*\mu 0} \equiv T^{*\mu 0}; \mu = 0 \ . \]

From \( \nabla_a T^{*bc} = \partial_a T^{*bc} + \Gamma^b_{ad} T^{*dc} + \Gamma^c_{ad} T^{*bd} \), we obtain

\[ T^{*00};0 = \partial_t T^{*00} + 2 \Gamma^0_{00} T^{*00} = \frac{\dot{\rho}(t)}{c^2(t)} , \]

\[ T^{*10};1 = \Gamma^0_{01} T^{*00} + \Gamma^0_{11} T^{*11} = \left[ \rho(t) + \frac{P(t)}{c^2(t)} \right] \frac{a(t)}{a(t)} \frac{1}{c^2(t)} , \]

\[ T^{*20};2 = \Gamma^2_{02} T^{*00} + \Gamma^2_{22} T^{*22} = \left[ \rho(t) + \frac{P(t)}{c^2(t)} \right] \frac{a(t)}{a(t)} \frac{1}{c^2(t)} , \]

\[ T^{*30};3 = \Gamma^3_{03} T^{*00} + \Gamma^3_{33} T^{*33} = \left[ \rho(t) + \frac{P(t)}{c^2(t)} \right] \frac{a(t)}{a(t)} \frac{1}{c^2(t)} . \]

Therefore

\[ T^{*\mu 0};\mu = \frac{\dot{\rho}(t)}{c^2(t)} + 3 \left[ \rho(t) + \frac{P(t)}{c^2(t)} \right] \frac{a(t)}{a(t)} \frac{1}{c^2(t)} , \]

which, together with \( T^{*\mu 0};\mu = 0 \), yields

\[ \dot{\rho}(t) + 3 \left[ \rho(t) + \frac{P(t)}{c^2(t)} \right] \frac{a(t)}{a(t)} = 0 . \]

Thus, for a universe composed of pressure-free dust \((P = 0)\) we have

\[ \dot{\rho}(t) + 3 \rho(t) \frac{a(t)}{a(t)} = 0 \]

\[ \Rightarrow \dot{\rho}(t)a^3(t) + 3 \rho(t) a^2(t) \frac{a(t)}{a(t)} = 0 \]
\[ \Rightarrow \frac{d}{dt} \left[ \rho(t) a^3(t) \right] = 0 , \]

or equivalently,

\[ \rho(t) a^3(t) = \text{constant} ; \] \hspace{1cm} (A2.1)

while for a universe composed of dust and radiation \((P = \rho c^2 / 3)\) we obtain

\[ \dot{\rho}(t) + 4 \rho(t) \frac{\dot{a}(t)}{a(t)} = 0 \]

\[ \Rightarrow \dot{\rho}(t) a^4(t) + 4 \rho(t) a^3(t) \dot{a}(t) = 0 \]

\[ \Rightarrow \frac{d}{dt} \left[ \rho(t) a^4(t) \right] = 0 , \]

thus

\[ \rho(t) a^4(t) = \text{constant} . \] \hspace{1cm} (A2.2)

3. The evolution equations for the universe

Plugging the expression for \(R\) and those for the components of \(R_{ab}, g_{ab}\) and \(T^*_{ab}\) into the field equation

\[ R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T^*_{ab} , \]

since the three spatial field equations are equivalent due to the homogeneity and isotropy, we obtain just two equations:

\[ \left[ \frac{\dot{a}(t)}{a(t)} \right]^2 = \left[ \frac{8\pi}{3} \rho(t) - \frac{1}{a^2(t)} \right] c^2(t) , \text{and} \] \hspace{1cm} (A3.1)

\[ \left[ \frac{\ddot{a}(t)}{a(t)} - 2 \frac{\dot{a}(t)}{a(t)} \frac{\dot{c}(t)}{c(t)} \right]^2 + 2 \frac{\ddot{a}(t)}{a(t)} - 2 \frac{\dot{a}(t)}{a(t)} \frac{\dot{c}(t)}{c(t)} = \left( 8\pi \frac{P(t)}{c^2(t)} + \frac{1}{a^2(t)} \right) c^2(t) . \] \hspace{1cm} (A3.2)
Using equation (A3.1), we may rewrite equation (A3.2) as

$$\frac{\ddot{a}(t)}{a(t)} - \frac{\dot{a}(t)}{a(t)} \frac{\dot{c}(t)}{c(t)} = -\frac{4\pi}{3} \left( \rho(t) + 3 \frac{p(t)}{c^2(t)} \right) c^2(t) .$$

Repeating the calculation for the cases of spatially flat and hyperboloid geometries, we obtain the general evolution equations for homogeneous, isotropic universe:

$$\left[ \frac{\ddot{a}(t)}{a(t)} \right]^2 = \left[ \frac{8\pi}{3} \rho(t) - \frac{k}{a^2(t)} \right] c^2(t) , \text{ and} \quad (A3.3)$$

$$\frac{\ddot{a}(t)}{a(t)} - \frac{\dot{a}(t)}{a(t)} \frac{\dot{c}(t)}{c(t)} = -\frac{4\pi}{3} \left( \rho(t) + 3 \frac{p(t)}{c^2(t)} \right) c^2(t) ,$$

where \( k = 1 \) for the 3-sphere, \( k = 0 \) for flat space, and \( k = -1 \) for the hyperboloid.

Equation (A3.3) can be rewritten as

$$\left[ \frac{\ddot{a}(t)}{c(t)} \right]^2 = \frac{8\pi}{3} \rho(t) a^2(t) - k .$$

Using equation (A2.1) or, respectively, equation (A2.2), we obtain, for a universe composed of dust only,

$$\left[ \frac{\ddot{a}(t)}{a(t)} \right]^2 = \frac{2M}{a(t)} - k ,$$

where \( M \equiv 4\pi\rho(t)a^3(t)/3 \), which is constant by equation (A2.1); while for a universe composed of dust and radiation,

$$\left[ \frac{\ddot{a}(t)}{a(t)} \right]^2 = \frac{2M'}{a^2(t)} - k ,$$

where \( M' \equiv 4\pi\rho(t)a^4(t)/3 \), which is constant by equation (A2.2).
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